Electromagnetic interactions of particles with matter

February 27, 2006

Abstract

This document is a brief review to the main mechanisms of electromagnetic interactions of charged particles and photons with matter, pertinent in Particle Physics, and their inplementation in GEANT4.

'Standard' em physics : the model

The projectile is assumed to have an energy ≥ 1 keV.

- The atomic electrons are quasi-free : their binding energy is neglected (except for photoelectric effect).
- The atomic nucleus is fixe : the recoil momentum is neglected.

The matter is described as homogeneous, isotropic, amorphous.

1. Common to all charged particles
   - ionization ($\sim keV \longrightarrow$)
   - Coulomb scattering from nuclei ($\sim keV \longrightarrow$)
   - Cerenkov effect
   - Scintillation
   - transition radiation
2. Muons
   - (e+,e-) pair production ($\sim 100GeV \longrightarrow$)
   - bremsstrahlung ($\sim 100GeV \longrightarrow$)
   - nuclear interaction ($\sim 1TeV \longrightarrow$)
3. Electrons and positrons
   - bremsstrahlung ($\sim 10MeV \longrightarrow$)
   - e+ annihilation

4. Photons
   - gamma conversion ($\sim 10MeV \longrightarrow$)
   - incoherent scattering ($\sim 100keV \longrightarrow \sim 10MeV$)
   - photo electric effect ($\longleftarrow \sim 100keV$)
   - coherent scattering ($\longleftarrow \sim 100keV$)
5. Optical photons
   - reflection and refraction
   - absorption
   - Rayleigh scattering

Total :$\sim$ 15 processes $\longrightarrow$ $\sim$ 40 classes

+ $\sim$ 10 classes for the materials category

A few words about the GEANT4 processes in general

A process may have three types of actions :

- well located in space : PostStep action
- not well located in space : AlongStep action
- well located in time : AtRest action

Each action is twofold :

- predicts where/when the interaction will occur : `GetPhysicalInteractionLength()`
- computes the final state of the interaction, where/when it occurs : `DoIt()`

A process has to fill 1, 2 or 3 couples of the following methods :

	AtRest	AlongStep	PostStep
GetPhysicalInteractionLength()			
DoIt()			

- DiscreteProcess is shortcut for a process which have **only** PostStep action.
- ContinuousProcess is shortcut for a process which have **only** AlongStep action.
- AtRestProcess is shortcut for a process which have **only** AtRest action.

examples

- discrete process : Compton scattering
  step determined by cross section, interaction at the end of the step (PostStepAction).
- continuous process : Cerenkov effect
  photons are created along the step, nb of photons (roughly) proportional to the step length (AlongStepAction).
- at rest process : no displacement, time is the relevant variable, e.g. positron annihilation at rest.

These are the 'pure' process types.

Some of the e.m. processes have combinations of actions :

- ionisation : continuous (energy loss) + discrete (Moller/Bhabha scattering, knock-on electron production)
- bremsstrahlung : continuous (energy loss due to soft photons) + discrete (hard photon emission)

in both cases the production threshold separates the continuous and discrete part of the process :

- if the (kinetic) energy of the secondary $\leq$ threshold energy, the secondary is not created , the effect of these soft interactions are treated as a continuous energy loss
- if the energy of the secondary is big enough, it is created at the end of the step (discrete part)

PhysicsList

For each type of particle the ProcessManager maintains a list of processes to be apply.

More precisely, there are 3 ordered lists of processes :

- AtRest action
- AlongStep action
- PostStep action

These lists are registered in the `UserPhysicsList` class.

example of PhysicsList

```
if (particleName == "e-") {
pmanager->AddProcess(new G4MultipleScattering, -1, 1,1);
pmanager->AddProcess(new G4eIonisation,        -1, 2,2);
pmanager->AddProcess(new G4eBremsstrahlung,    -1,-1,3);
}

else if (particleName == "e+") {
pmanager->AddProcess(new G4MultipleScattering, -1, 1,1);
pmanager->AddProcess(new G4eIonisation,        -1, 2,2);
pmanager->AddProcess(new G4eBremsstrahlung,    -1,-1,3);
pmanager->AddProcess(new G4eplusAnnihilation,   0,-1,4);
}
```

```
if (particleName == "mu+" || particleName == "mu-") {
pmanager->AddProcess(new G4MultipleScattering, -1, 1,1);
pmanager->AddProcess(new G4MuIonisation,       -1, 2,2);
pmanager->AddProcess(new G4MuBremsstrahlung,   -1,-1,3);
pmanager->AddProcess(new G4MuPairProduction,   -1,-1,4);
}

if ((particle->GetPDGCharge() != 0.0) &&
(!particle->IsShortLived()) &&
(particle->GetParticleName() != "chargedgeantino")) {
pmanager->AddProcess(new G4MultipleScattering, -1,1,1);
pmanager->AddProcess(new G4hIonisation,        -1,2,2);
}
```

```
if (particleName == "gamma") {
pmanager->AddDiscreteProcess(new G4PhotoElectricEffect);
pmanager->AddDiscreteProcess(new G4ComptonScattering);
pmanager->AddDiscreteProcess(new G4GammaConversion);
}
```

is a shortcut for :

```
pmanager->AddProcess(new G4PhotoElectricEffect, -1,-1,1);
pmanager->AddProcess(new G4ComptonScattering,   -1,-1,2);
pmanager->AddProcess(new G4GammaConversion,     -1,-1,3);
```

For processes which have only PostStepAction, the ordering is not important.

Compton scattering

The Compton effect describes the scattering off quasi-free atomic electrons :

$$\gamma + e \rightarrow \gamma' + e'$$

Each atomic electron acts as an independent cible; Compton effect is called incoherent scattering. Thus:

$$\text{cross section per atom} = Z \times \text{cross section per electron}$$

The inverse Compton scattering also exists: an energetic electron collides with a low energy photon which is blue-shifted to higher energy. This process is of importance in astrophysics.

Compton scattering is related to (e^+, e^-) annihilation by crossing symmetry.

energy spectrum

Under the same assumption,the unpolarized differential cross section per atom is given by the Klein-Nishina formula [Klein29] :

$$\frac{d\sigma}{dk'} = \frac{\pi\, r_e^2}{mc^2}\,\frac{Z}{\kappa^2}\,\left[\epsilon + \frac{1}{\epsilon} - \frac{2}{\kappa}\,\left(\frac{1-\epsilon}{\epsilon}\right) + \frac{1}{\kappa^2}\,\left(\frac{1-\epsilon}{\epsilon}\right)^2\right] \qquad (1)$$

where

k'	energy of the scattered photon ; $\epsilon = k'/k$
r_e	classical electron radius
κ	k/mc^2

total cross section per atom

$$\sigma(k) = \int_{k'_{min}=k/(2\kappa+1)}^{k'_{max}=k} \frac{d\sigma}{dk'}\, dk'$$

$$\sigma(k) = 2\pi\, r_e^2\, Z\, \left[\left(\frac{\kappa^2 - 2\kappa - 2}{2\kappa^3} \right) \ln(2\kappa+1) + \frac{\kappa^3 + 9\kappa^2 + 8\kappa + 2}{4\kappa^4 + 4\kappa^3 + \kappa^2} \right]$$

limits

$k \rightarrow \infty :$ σ goes to 0 : $\sigma(k) \sim \pi\, r_e^2\, Z\, \dfrac{\ln 2\kappa}{\kappa}$

$k \rightarrow 0 :$ $\sigma \rightarrow \dfrac{8\pi}{3}\, r_e^2\, Z$ (classical Thomson cross section)

low energy limit

In fact, when $k \leq 100\ keV$ the binding energy of the atomic electron must be taken into account by a corrective factor to the Klein-Nishina cross section:

$$\frac{d\sigma}{dk'} = \left[\frac{d\sigma}{dk'}\right]_{KN} \times S(k, k')$$

See for instance [Cullen97] or [Salvat96] for derivation(s) and discussion of the *scattering function* S(k,k').

As a consequence, at very low energy, the total cross section goes to 0 like k^2. It also suppresses the forward scattering.

At X-rays energies the scattering function has little effect on the Klein-Nishina energy spectrum formula 1. In addition the Compton scattering is not the dominant process in this energy region.

total cross section per atom in GEANT4

The total cross section has been parametrized :

$$\sigma(Z,\kappa) = \left[P_1(Z)\ \frac{\log(1+2\kappa)}{\kappa} + \frac{P_2(Z) + P_3(Z)\kappa + P_4(Z)\kappa^2}{1 + a\kappa + b\kappa^2 + c\kappa^3} \right]$$

where:

$$\begin{aligned} \kappa &= k/mc^2 \\ P_i(Z) &= Z(d_i + e_i Z + f_i Z^2) \end{aligned}$$

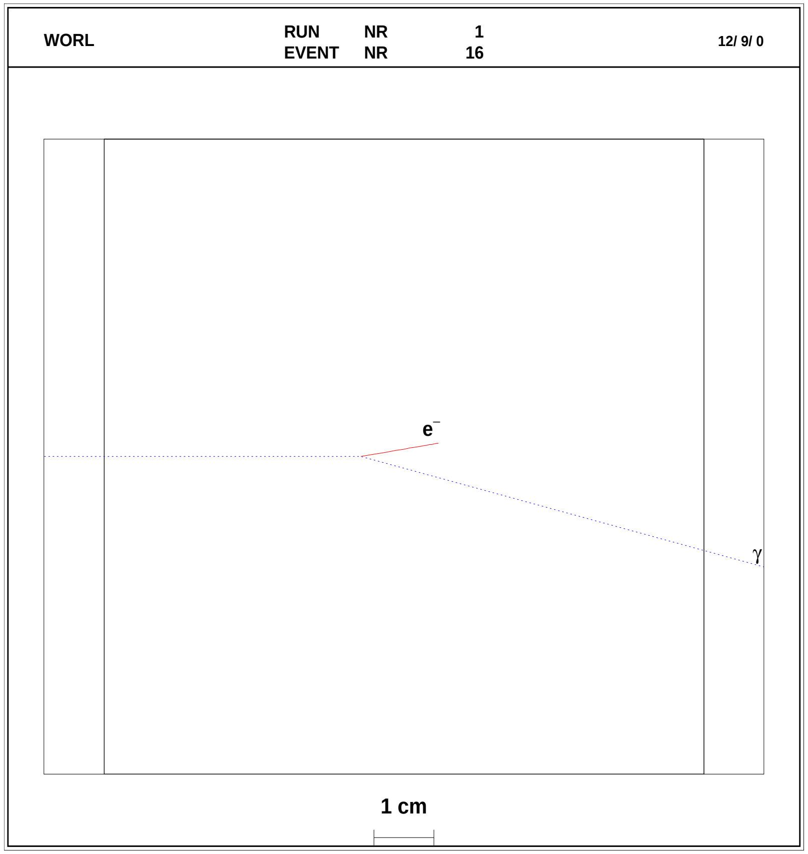
The fit was made over 511 data points chosen between:

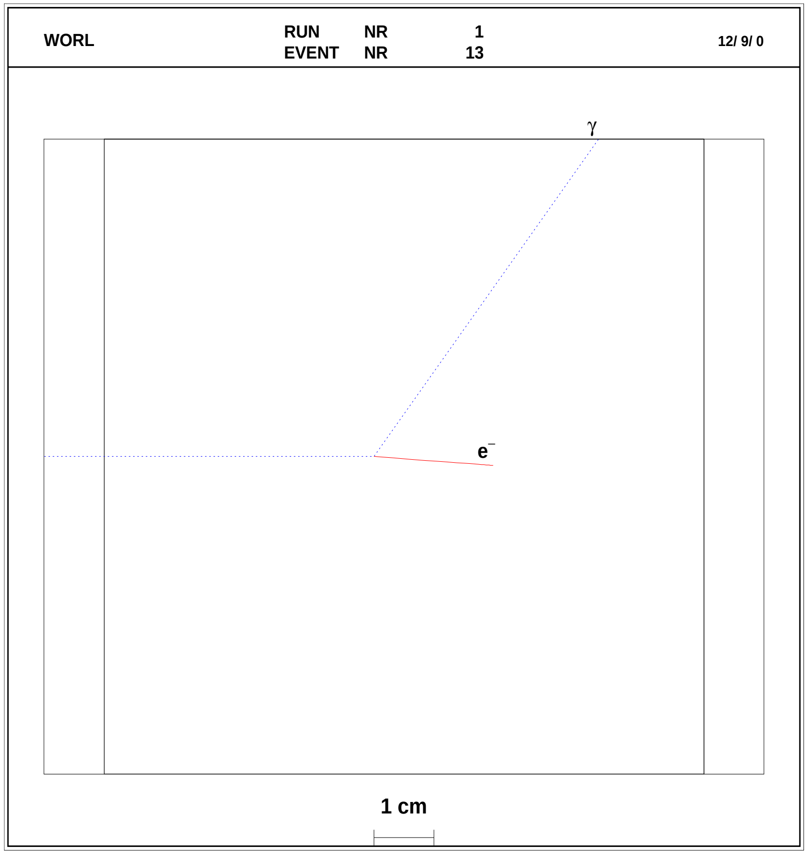
$$1 \leq Z \leq 100 \qquad ; \qquad k \in [10\ \text{keV}, 100\ \text{GeV}]$$

The accuracy of the fit is estimated to be:

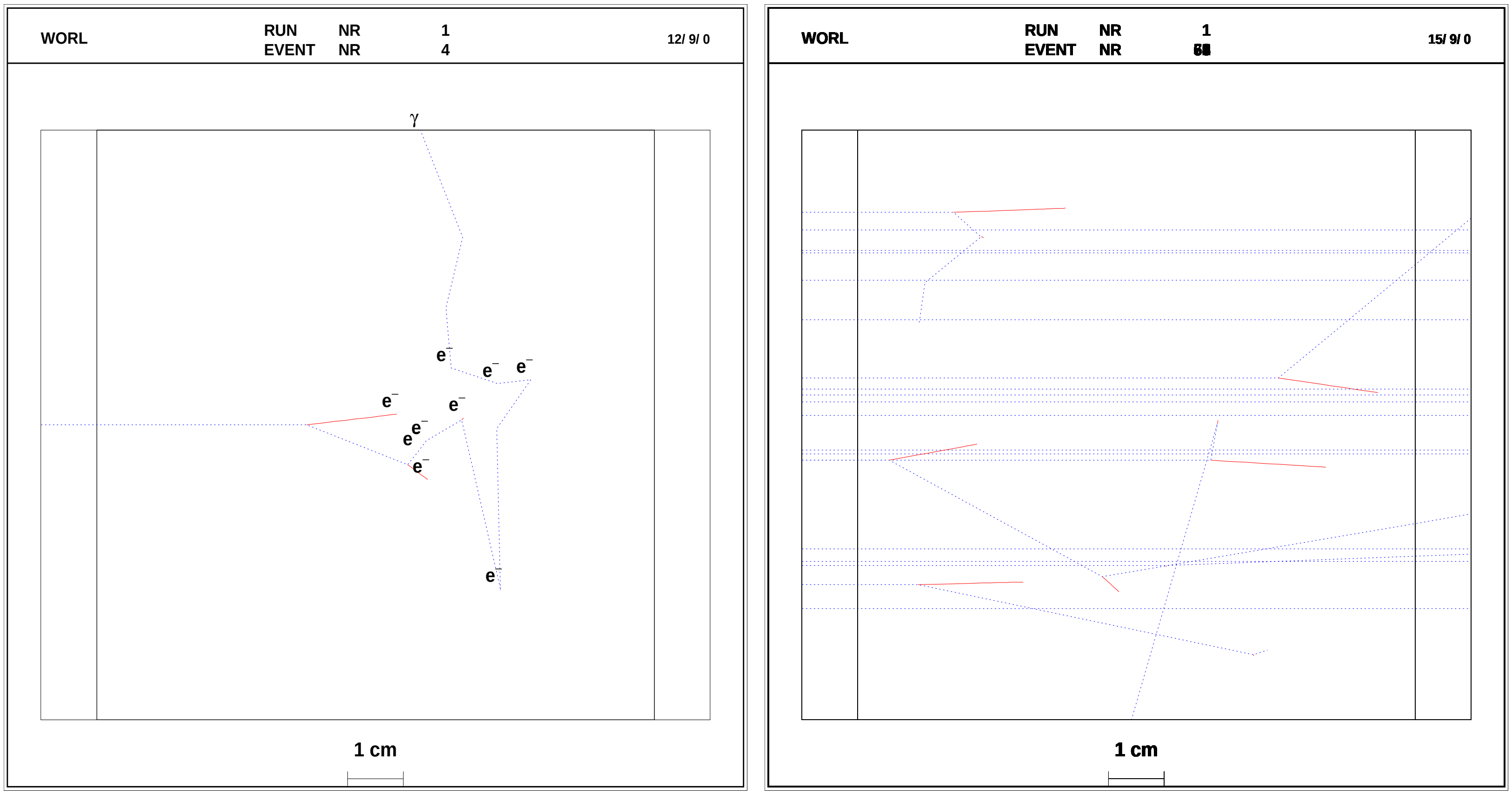
$$\frac{\Delta\sigma}{\sigma} = \begin{cases} \approx 10\% & \text{for } k \simeq 10\ \text{keV} - 20\ \text{keV} \\ \leq 5 - 6\% & \text{for } k > 20\ \text{keV} \end{cases}$$

γ 10 MeV in 10 cm Aluminium: Compton scattering





γ 10 MeV in 10 cm Aluminium: Compton scattering



Ionization

The basic mechanism is an inelastic collision of the moving charged particle with the atomic electrons of the material, ejecting off an electron from the atom :

$$\mu + atom \rightarrow \mu + atom^{+} + e^{-}$$

In each individual collision, the energy transfered to the electron is small. But the total number of collisions is large, and we can well define the average energy loss per (macroscopic) unit path length.

Mean energy loss and energetic δ-rays

$$\frac{d\sigma(Z,E,T)}{dT}$$

is the differential cross-section per atom for the ejection of an electron with kinetic energy T by an incident charged particle of total energy E moving in a material of density ρ.

One may wish to take into account separately the high-energy knock-on electrons produced above a given threshold T_{cut} (miss detection, explicit simulation ...).

$T_{cut} \gg I$ (mean excitation energy in the material).

$T_{cut} > 1$ keV in GEANT4

Below this threshold, the soft knock-on electrons are counted only as continuous energy lost by the incident particle.

Above it, they are explicitly generated. Those electrons must be excluded from the mean continuous energy loss count.

The mean rate of the energy lost by the incident particle due to the soft δ-rays is :

$$\frac{dE_{soft}(E, T_{cut})}{dx} = n_{at} \cdot \int_{0}^{T_{cut}} \frac{d\sigma(Z, E, T)}{dT} T \, dT \qquad (2)$$

n_{at} : nb of atoms per volume in the matter.

The total cross-section per atom for the ejection of an electron of energy $T > T_{cut}$ is :

$$\sigma(Z, E, T_{cut}) = \int_{T_{cut}}^{T_{max}} \frac{d\sigma(Z, E, T)}{dT} \, dT \qquad (3)$$

where T_{max} is the maximum energy transferable to the free electron.

Mean rate of energy loss by heavy particles

The integration of 2 leads to the well known Bethe-Bloch truncated energy loss formula [PDG] :

$$\left.\frac{dE}{dx}\right]_{T<T_{cut}} = 2\pi r_e^2 mc^2 n_{el} \frac{(z_p)^2}{\beta^2} \times \left[\ln\left(\frac{2mc^2\beta^2\gamma^2 T_{up}}{I^2}\right) - \beta^2\left(1 + \frac{T_{up}}{T_{max}}\right) - \delta - \frac{2C_e}{Z}\right]$$

Fluctuations in energy loss

$\langle \Delta E \rangle = (dE/dx).\Delta x$ gives only the average energy loss by ionization. There are fluctuations. Depending of the amount of matter in Δx the distribution of ΔE can be strongly asymmetric ($\rightarrow$ the Landau tail).

The large fluctuations are due to a small number of collisions with large energy transfers.

Energy loss fluctuations : the model in GEANT

Based on a very simple model of the particle-atom interaction.

The atoms are assumed to have only two energy levels E_1 and E_2.

The particle-atom interaction can be :

- an excitation with energy loss E_1 or E_2
- an ionization with energy loss distribution $g(E) \sim 1/E^2$.

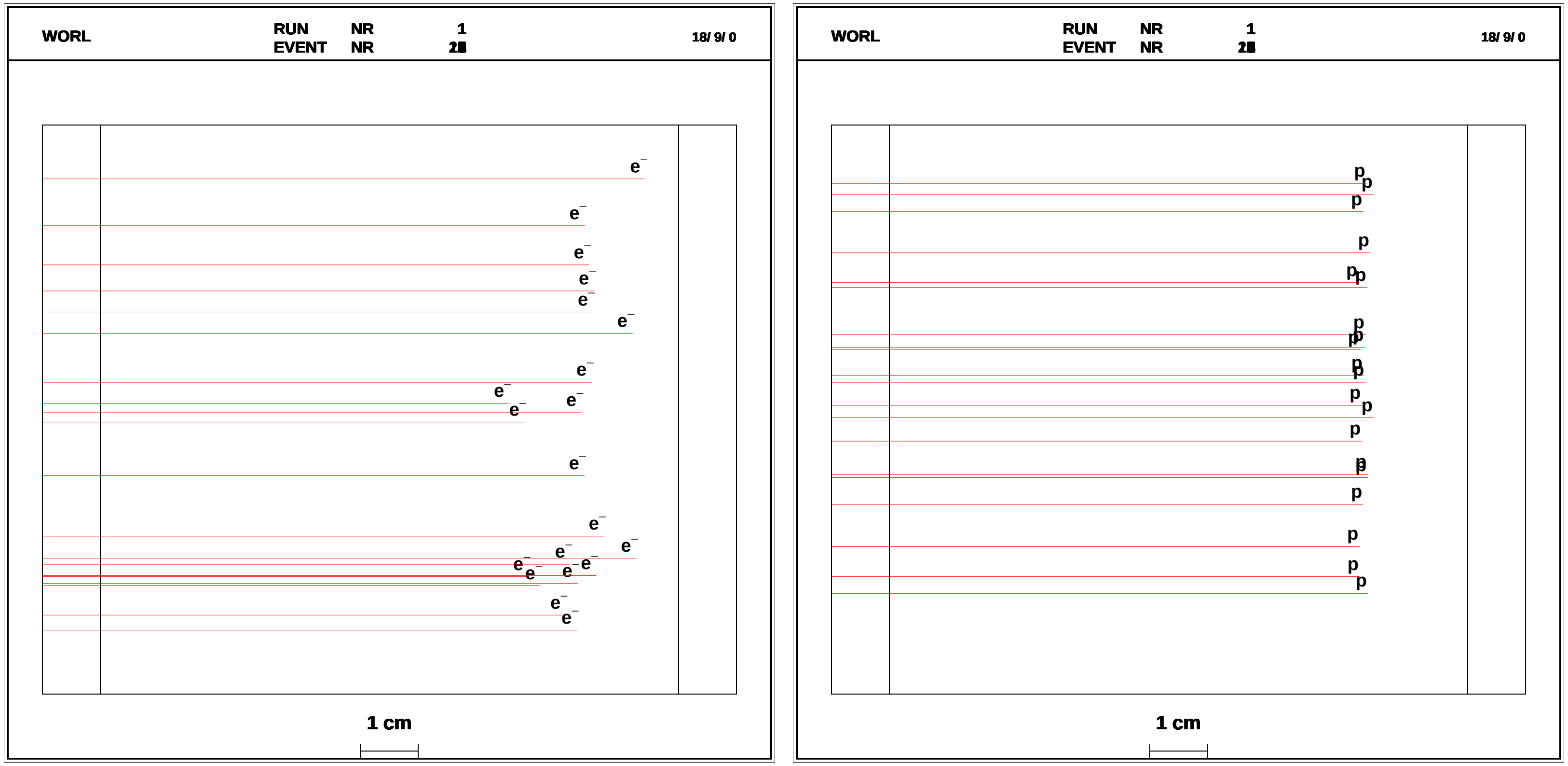
This simple model of the energy loss fluctuations is rather fast and it can be used for any thickness of the materials, and for any T_{cut}.

This has been proved performing many simulations and comparing the results with experimental data, see e.g [Urban95].

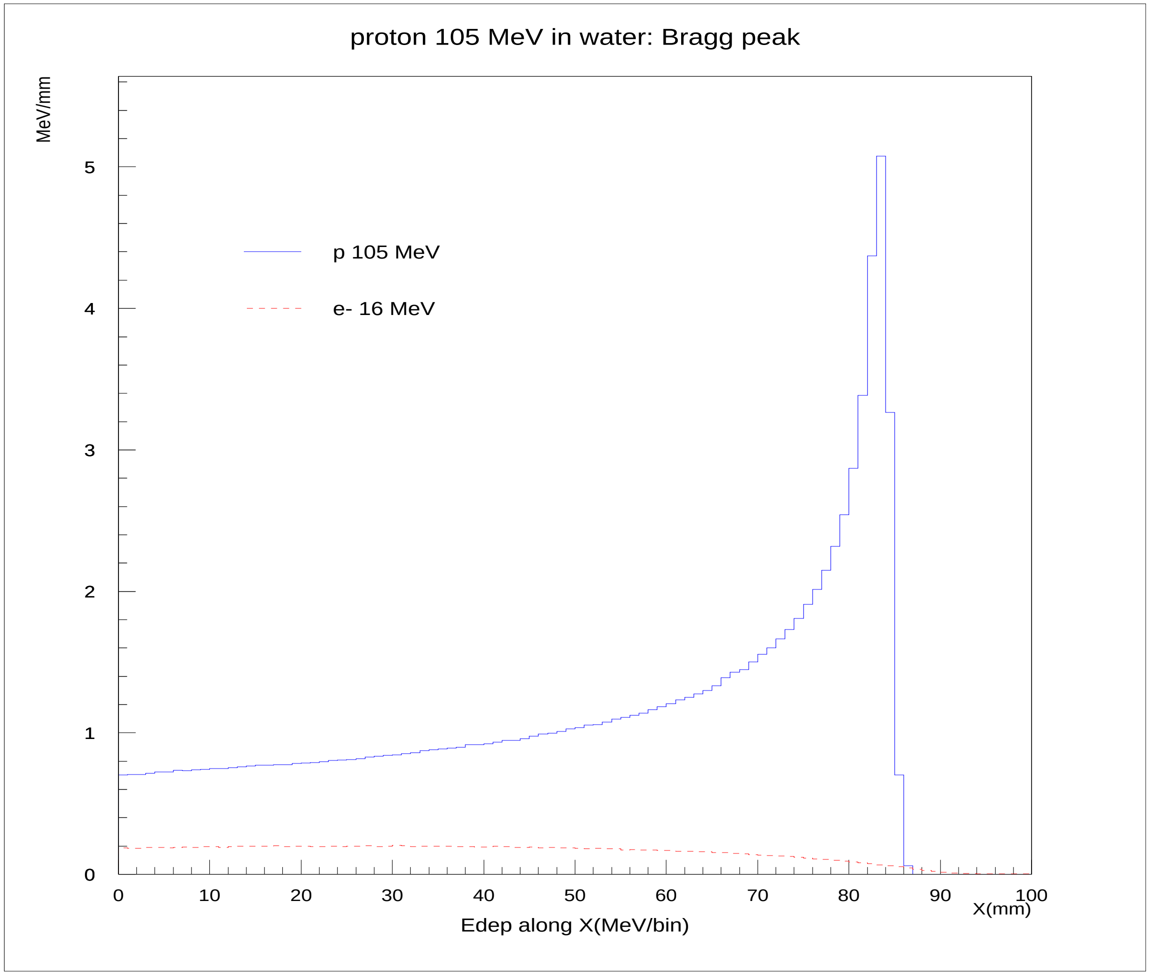
Approaching the limit of the validity of Landau's theory, the loss distribution approaches smoothly the Landau form.

Fluctuations on ΔE lead to fluctuations on the actual range (straggling).

penetration of e^- (16 MeV) and proton (105 MeV) in 10 cm of water.



Bragg curve. More energy per unit length are deposit towards the end of trajectory rather at its beginning.



Energetic δ rays

The differential cross-section per atom for producing an electron of kinetic energy T, with $I \ll T_{cut} \leq T \leq T_{max}$, can be written :

$$\frac{d\sigma}{dT} = 2\pi r_e^2 mc^2 Z \frac{z_p^2}{\beta^2} \frac{1}{T^2} \left[1 - \beta^2 \frac{T}{T_{max}} + \frac{T^2}{2E^2} \right]$$

(the last term for spin 1/2 only).

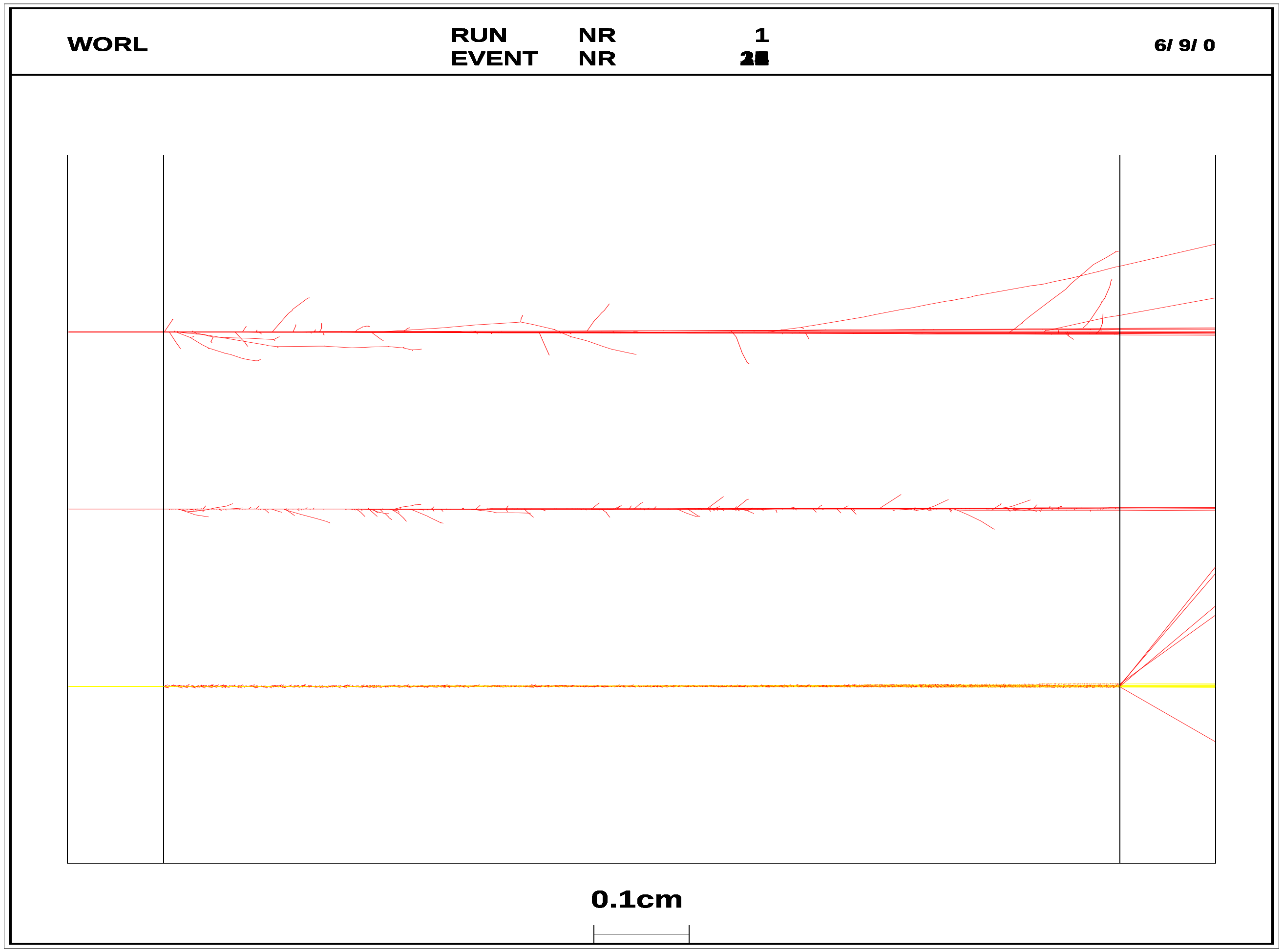
The integration (3) gives :

$$\sigma(Z, E, T_{cut}) = \frac{2\pi r_e^2 Z z_p^2}{\beta^2} \left[\left(\frac{1}{T_{cut}} - \frac{1}{T_{max}} \right) - \frac{\beta^2}{T_{max}} \ln \frac{T_{max}}{T_{cut}} + \frac{T_{max} - T_{cut}}{2E^2} \right]$$

(the last term for spin 1/2 only).

delta rays

200 MeV electrons, protons, alphas in 1 cm of Aluminium



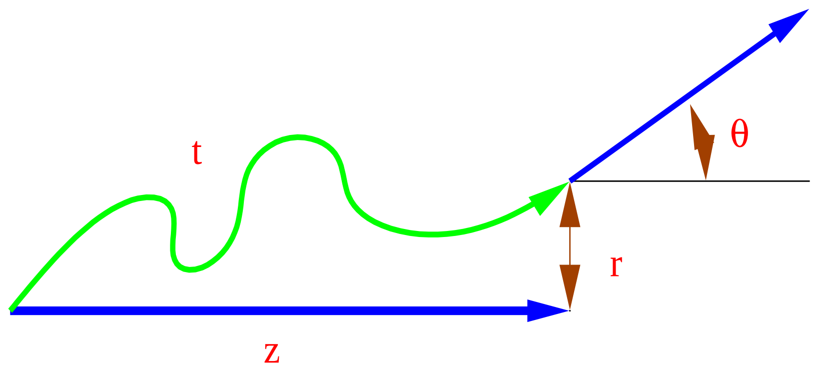
Incident electrons and positrons

For incident $e^{-/+}$ the Bethe Bloch formula must be modified because of the mass and identity of particles (for e^-).

One use the Moller or Bhabha cross sections [Mess70] and the Berger-Seltzer dE/dx formula [ICRU84, Selt84].

Multiple Coulomb scattering

Charged particles traversing a finite thickness of matter suffer repeated elastic Coulomb scattering. The cumulative effect of these small angle scatterings is a net deflection from the original particle direction.



- longitudinal displacement z (or geometrical path length)
- lateral displacement r, Φ
- true (or corrected) path length t
- angular deflection θ, ϕ

The practical solutions of the particle transport can be classified :

- detailed (microscopic) simulation : exact, but time consuming if the energy is not small. Used only for low energy particles.
- condensed simulation : simulates the global effects of the collisions during a macroscopic step, but uses approximations. EGS, Geant3 (both use Moliere theory), Geant4
- mixed algorithms : "hard collisions" are simulated one by one + global effects of the "soft collisions" : Penelope.

Angular distribution

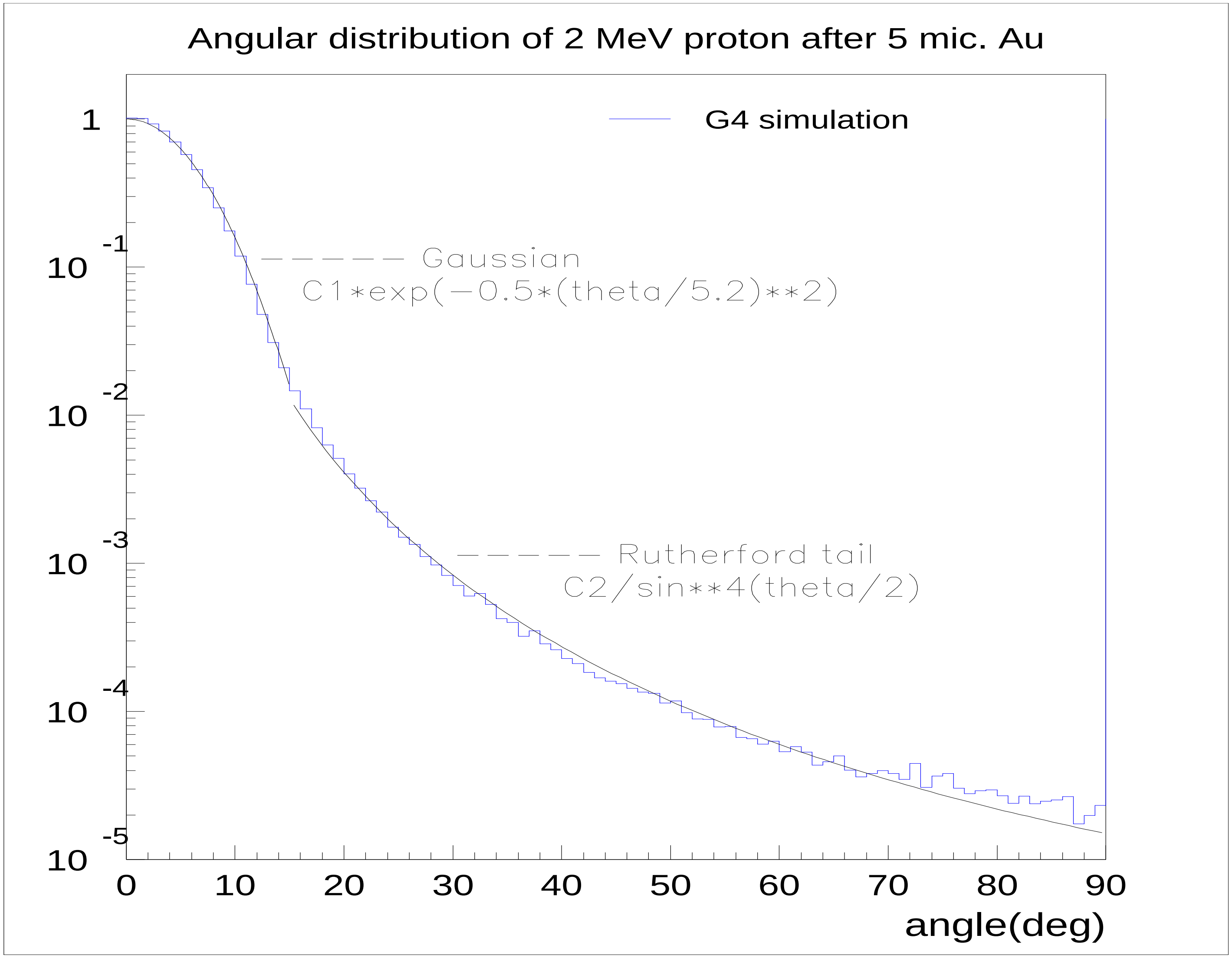
If the number of individual collisions is large enough (> 20) the multiple Coulomb scattering angular distribution is Gaussian at small angles and like Rutherford scattering at large angles.

The Molière theory [Mol48, Bethe53] reproduces rather well this distribution, but it is an approximation.

The Molière theory is accurate for not too low energy and for small angle scattering, but even for this case its accuracy is not too good for very low Z and high Z materials.(see e.g. [Fer93], [Gotts93])

The Molière theory does not give information about the spatial displacement of the particle, it gives only the scattering angle distribution.

Angular distribution



Gaussian approximation

The central part of the spatial angular distribution is approximately

$$P(\theta)\,d\Omega = \frac{1}{2\pi\theta_0^2}\ \exp\left[-\frac{\theta^2}{2\theta_0^2}\right]\ d\Omega$$

with

$$\theta_0 = \frac{13.6\ \mathrm{MeV}}{\beta\, pc} z \sqrt{\frac{l}{X_0}} \left[1 + 0.038 \ln\left(\frac{l}{X_0}\right)\right] \tag{4}$$

where l/X_0 is the thickness of the medium measured in radiation lengths X_0 ([Highl75],[Lynch91]).

This formula of θ_0 is from a fit to Molière distribution. It is accurate to $\leq 11\%$ for $10^{-3} < l/X_0 < 10^2$
This formula is used very often, but it is worth to note that this is an approximation of the Molière result for the small angle region with an error which can be as big as $\approx 10\%$.

To get a more complete information it is better to start with theory of Lewis which based on the transport equation of charged particles ([Lewis50, Kawrakow98]).

The MSC model in Geant4 uses model functions to sample angular and spatial distributions after a step.

The functions are choosen in such a way that they give the same moments than the Lewis theory.

The details of the MSC model can be found in the Geant4 Physics Reference Manual.

MSC Algorithm

Steps of MSC algorithm (are essentially the same for many condensed simulations) :

1. selection of step length $\Longleftarrow$ physics processes + geometry (MSC performs the $t \Longleftrightarrow z$ transformations only)
2. transport to the initial direction : (not MSC business)
3. sample scattering angle (θ, ϕ)
4. compute lateral displacement, relocate particle

Angle distributions central part + tail

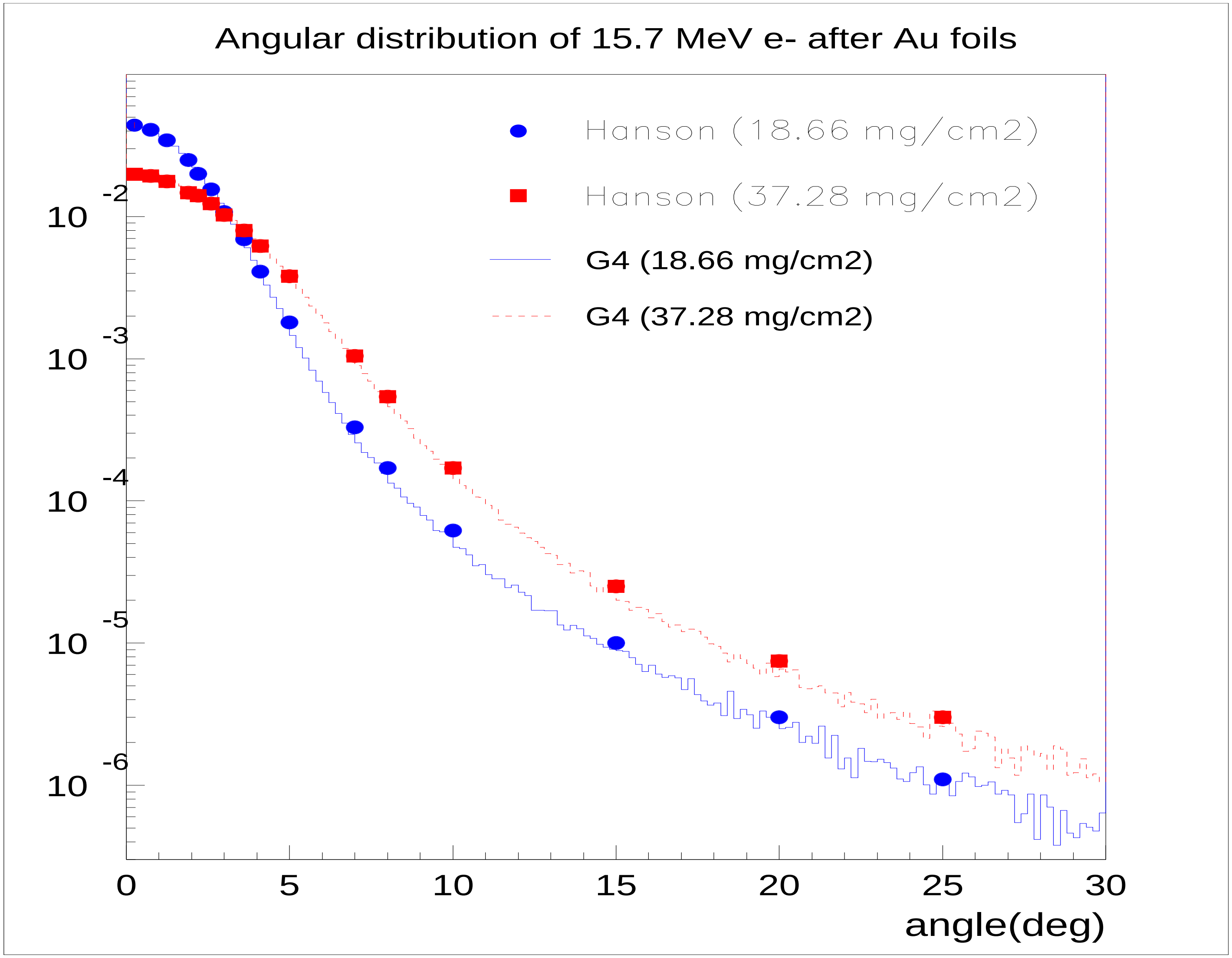
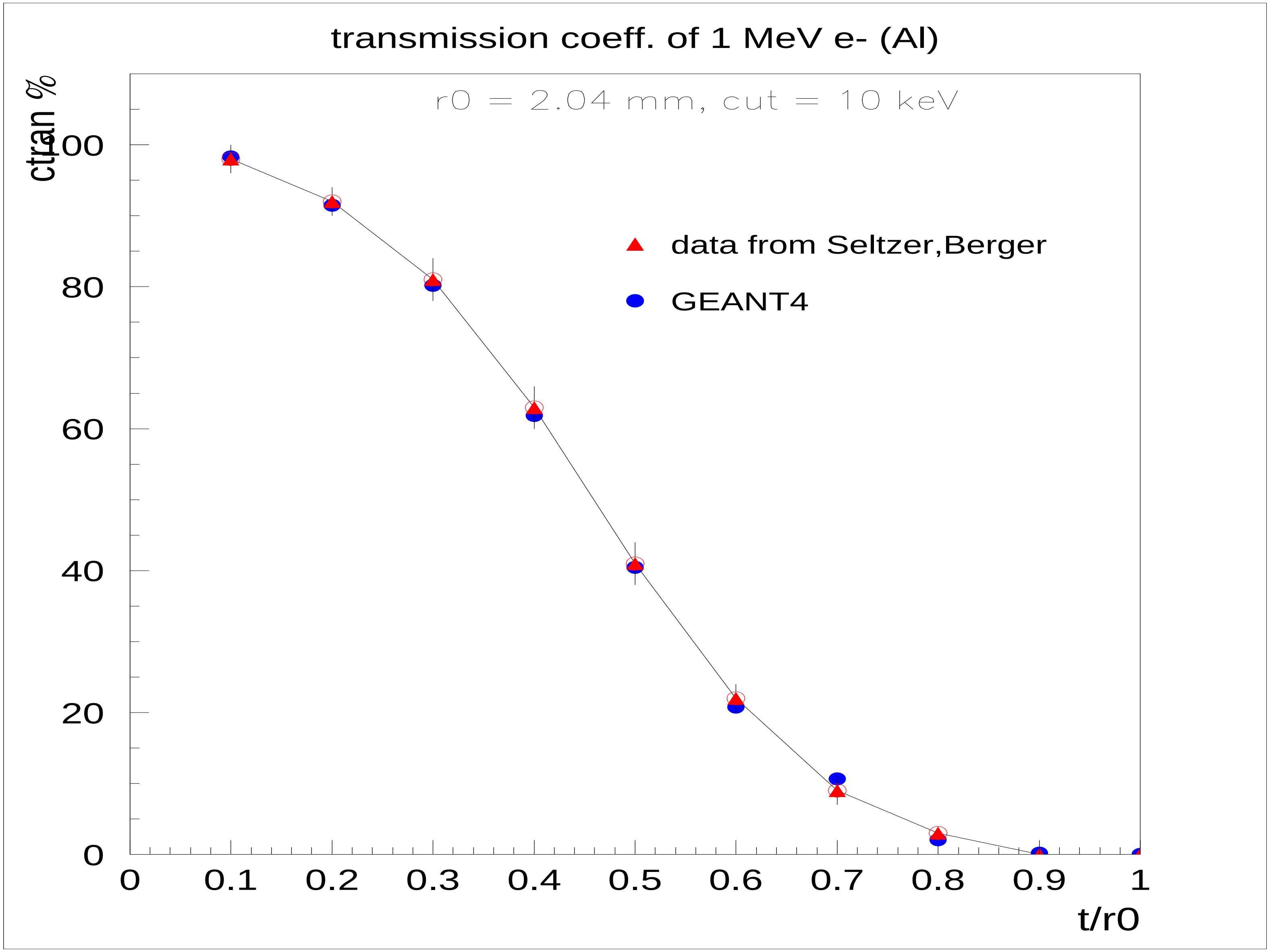
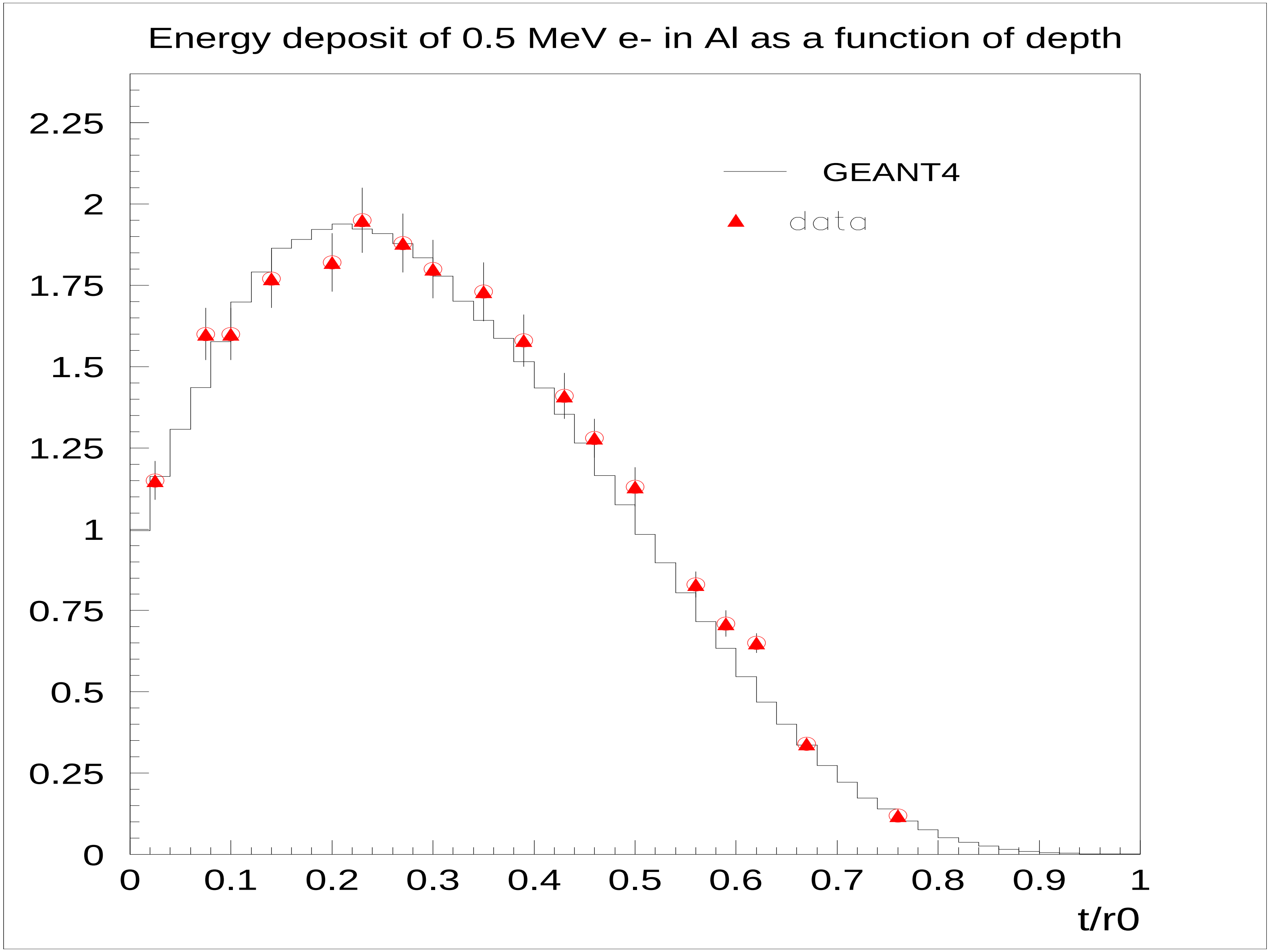


Fig. 3 shows the number transmission coefficient T as function of the foil thickness for 1 MeV electrons in aluminium. The thickness is measured in units of the continuous slowing down range, the data originated from different measurements have been taken from the review paper of Seltzer and Berger ([Seltz74]).


transmission coeff. of 1 MeV e- (Al)
r0 = 2.04 mm, cut = 10 keV
ctran %
t/r0
data from Seltzer,Berger
GEANT4
0
20
40
60
80
100
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


Energy deposit of 0.5 MeV e- in Al as a function of depth
2.25
2
1.75
1.5
1.25
1
0.75
0.5
0.25
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t/r0
GEANT4
data

Stepping/Boundary Crossing Algorithm

A very simple stepping/boundary crossing algorithm has been implemented in the MSC code for the backscattering:
At the start of a track or after entering in a new volume, restrict the step size :

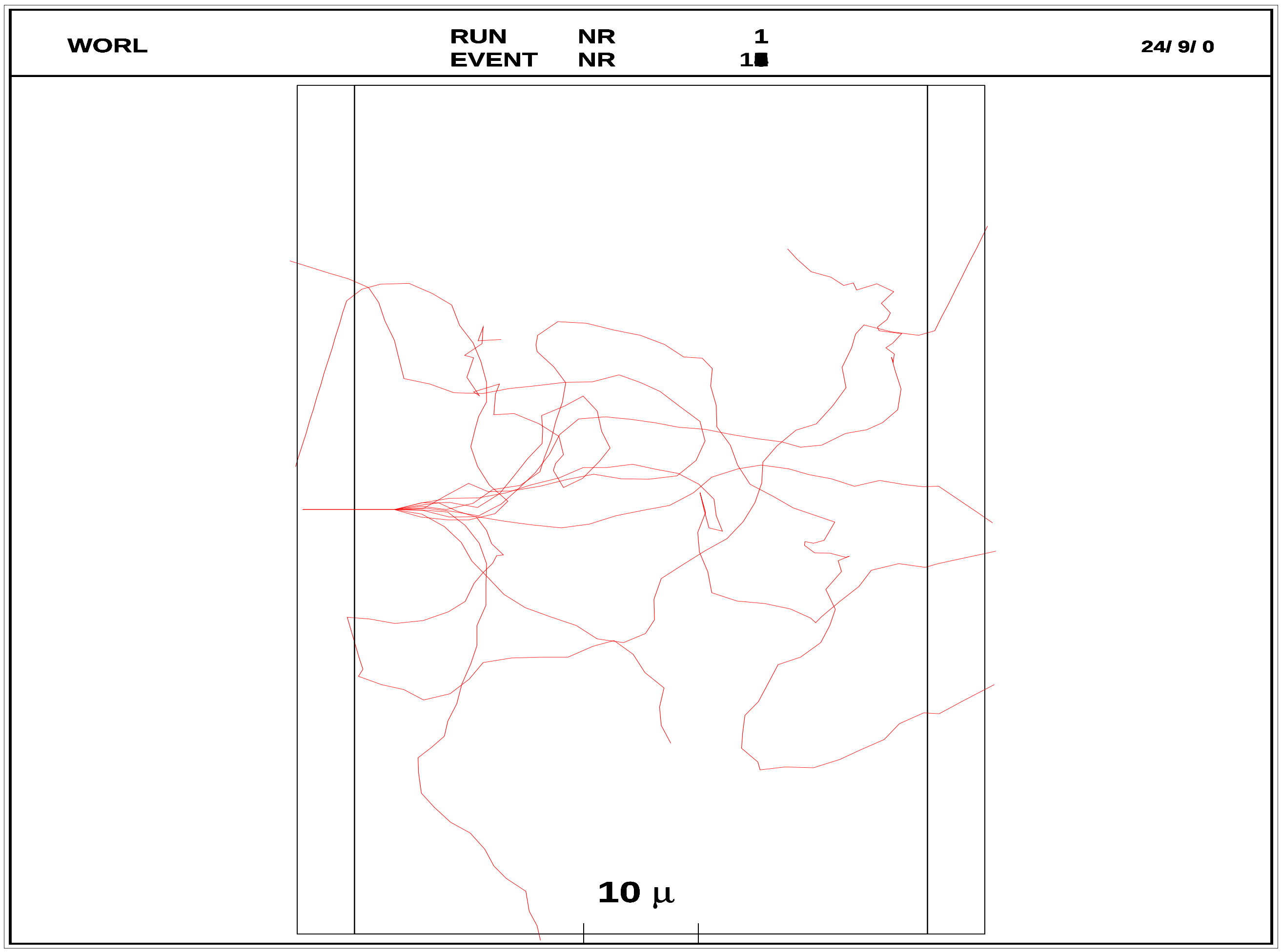
$$t = f_r \cdot max\{r, \lambda_1\} \tag{5}$$

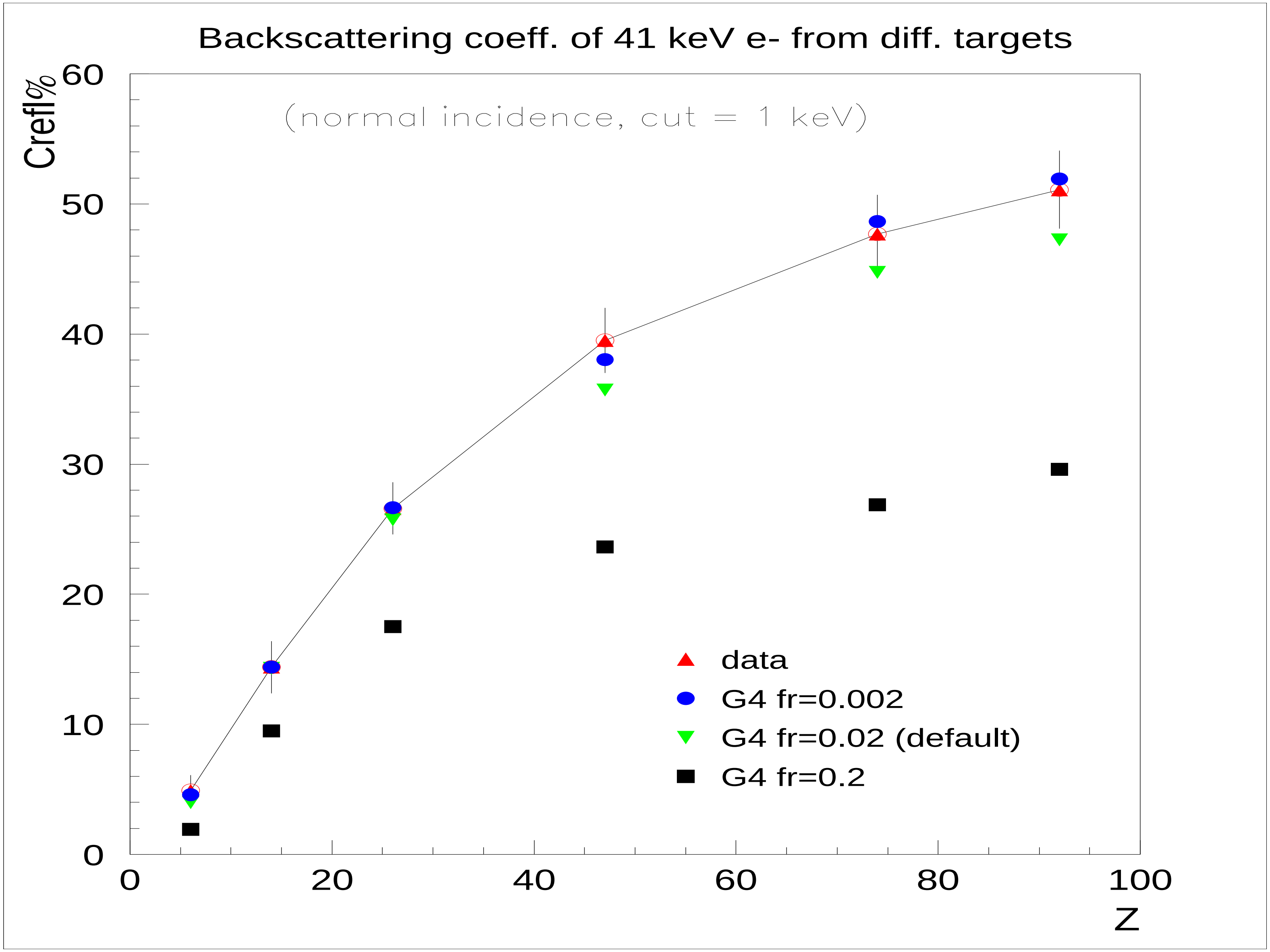
r is the range of the particle, f_r is a constant ($f_r \in [0, 1]$)
(taking the max of r and λ_1 is an empirical choice)

It can be easily seen that this kind of step limitation means a real contraints for low energy particles only.

backscattering of low energy electrons The incident beam is 10 electrons of 600 keV entering in 50 μm of Tungsten.

4 electrons are transmitted, 2 are backscattered.




Backscattering coeff. of 41 keV e- from diff. targets
(normal incidence, cut = 1 keV)
Crefl%
Z
0
10
20
30
40
50
60
0
20
40
60
80
100
data
G4 fr=0.002
G4 fr=0.02 (default)
G4 fr=0.2

Energy-Range relation

Mean total pathlength of a charged particle of kinetic energy E :

$$R(E) = \int_{\epsilon=0}^{\epsilon=E} \left[\frac{d\epsilon}{dx}\right]^{-1} d\epsilon$$

In GEANT4 the energy-range relation is extensively used :

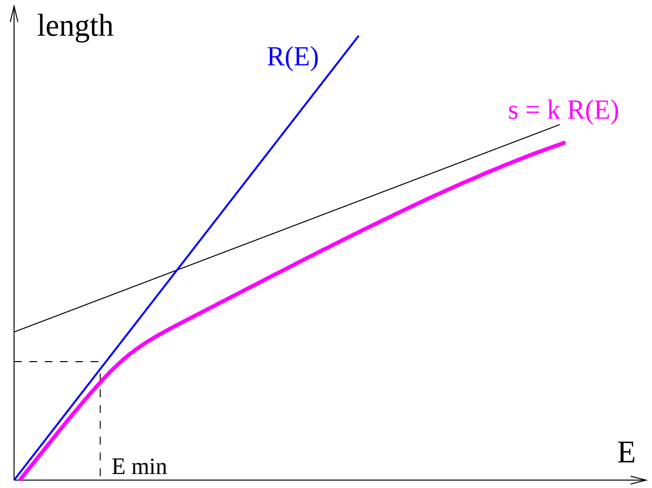
- to control the stepping of charged particles
- to compute the energy loss of charged particles
- to control the production of secondaries (cut in range)

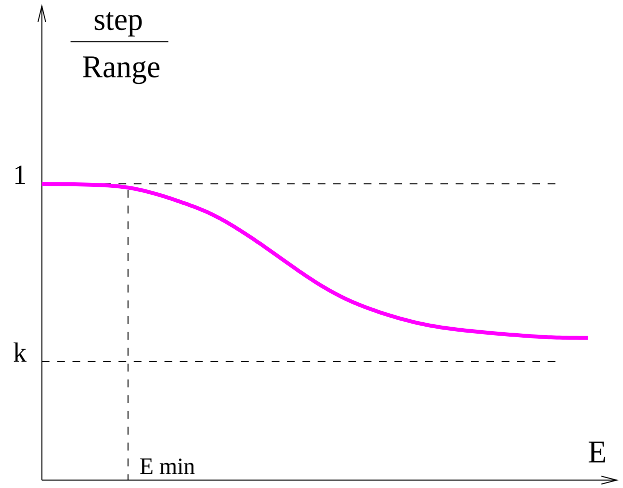
control the stepping of charged particles

The continuous energy loss imposes a limit on the stepsize.

The cross sections depend of the energy. The step size must be small enough so that the energy difference along the step is a small fraction of the particle energy.

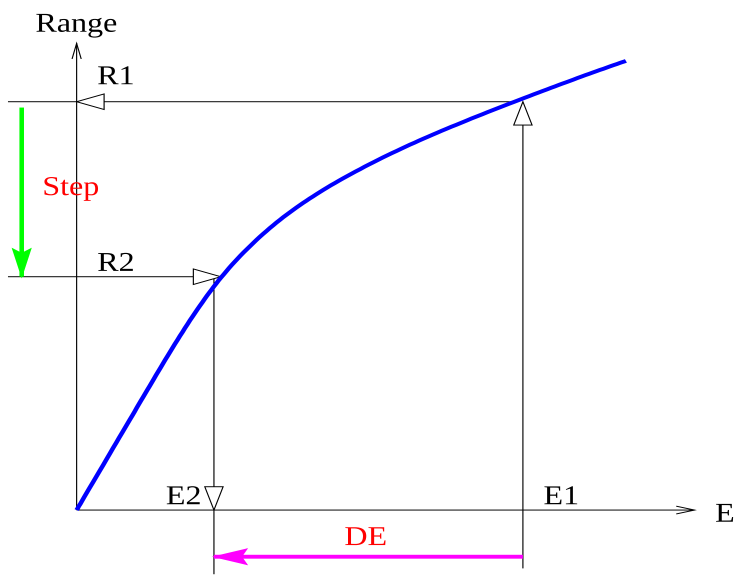
This constraint must be relaxed when $E \rightarrow 0$: the allowed step smoothly approaches the stopping range of the particle.





compute the mean energy loss of charged particles

The computation of the mean energy loss on a given step is done from the Range and inverse Range tables.



This is more accurate than $\Delta E = (dE/dx) * \text{stepLength}$.

On the same spirit, the time of life of the particle is updated from tables, automatically taking account that the particle velocity is slowing down along the step.

production thresholds (cuts) of secondaries

Production thresholds are expressed in range (instead in energy) for charged particles and photons (photon 'range' = abs.length)

No difference in a homogenous material, but GEANT4 choice is better in general, e.g. sampling calorimeter.

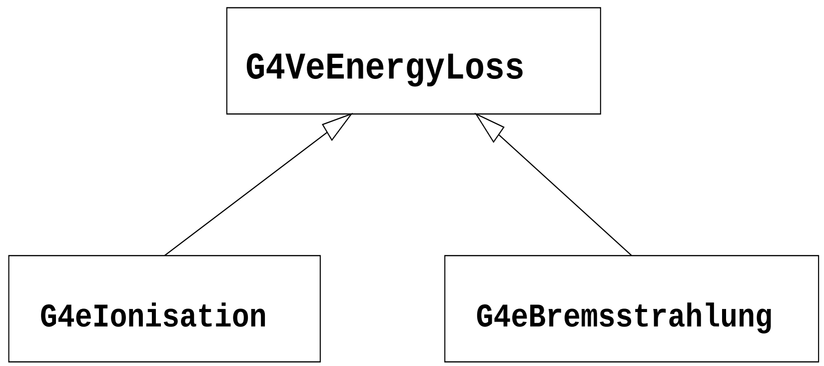
example : Pb + liquidArgon + Pb + liquid Argon
each layer is few mm thick → cut/threshold can be 1(0.1) mm,
cuts in energy $E_{lAr}^{cut} < E_{Pb}^{cut}$ give 'coherent' physics
while using the same energy cut in both material gives 'not so good' physics in the case of high cut or degrades the efficiency (speed) for small cut value.

design details

Ionization and Bremsstrahlung cannot be independent :

$$\left[\frac{dE}{dx}\right]_{tot} = \left[\frac{dE}{dx}\right]_{ioni} + \left[\frac{dE}{dx}\right]_{brem}$$

$$R(E) = \int_{\epsilon=0}^{\epsilon=E} \left[\frac{d\epsilon}{dx}\right]_{tot}^{-1} d\epsilon$$



The processes compute the individual contributions.

The base class computes the sum and the range.

The base class is pure virtual : it cannot be directly instantiated.

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