Direct \((e^+, e^-)\) pair creation by muon
Direct \((e^+, e^-)\) pair creation by muon

Creation of a \((e^+, e^-)\) pair by virtual photon in the Coulomb field of the nucleus (for momentum conservation).

\[
\mu + \text{nucleus} \longrightarrow \mu + e^+ + e^- + \text{nucleus}
\]
Direct \((e^+, e^-)\) pair creation by muon

It is one of the most important processes of muon interaction. At TeV muon energies, pair creation cross section exceeds those of other muon interaction processes in a wide region of energy transfers:

\[
100 \text{ MeV} \leq \epsilon \leq 0.1 \ E_\mu
\]

Average energy loss for pair production increases linearly with muon energy, and in TeV region this process contributes over 50% to the total energy loss rate.
energy transfers

Main contribution to the total cross section is given by transferred energies:

$$5\text{MeV} \leq \epsilon \leq 0.01 \ E_\mu$$

The contribution to average muon energy loss is determined mostly by region:

$$10^{-3} \ E_\mu \leq \epsilon \leq 0.1 \ E_\mu$$

Thus, to adequately describe the number of pairs produced, average energy loss and stochastic energy loss distribution one need to reproduce with a sufficient accuracy the differential cross section behaviour in a wide range of energy transfers:

$$5\text{MeV} \leq \epsilon \leq 0.1 \ E_\mu$$
Direct \((e^+, e^-)\) pair creation by muon

![Graph showing the distribution of \(e^+ e^-\) pairs, \(\delta\)-electrons, and bremsstrahlung as a function of muon energy.]
Direct $(e^+, e^-)$ pair creation by muon

![Graph showing energy loss ($dE/dx$) versus muon energy (GeV) for different processes involving iron (Fe). The graph includes lines for Fe total, Fe ion, Fe pair, Fe brems, and Fe nucl.]
differential cross section

The differential cross section is given by Kokoulin et al. [Koko71].

It includes:

- screening of the field of the nucleus
- correction for finite nuclear size
- contribution from the atomic electrons [Keln97]
- ...

See [Koko71] for a complete discussion.
differential cross section

The differential cross section per atom can be written as:

\[
\frac{d\sigma}{d\epsilon} = \frac{4 \alpha^2 r_e^2}{3\pi} \frac{1 - \nu}{\epsilon} [Z(Z + \zeta)] F(Z, E, \epsilon)
\]  

(1)

with

\[
F(Z, E, \epsilon) = \int_0^{\rho_{max}} \left[ \Phi_e(v, \rho) + \left( \frac{m_e}{m_\mu} \right)^2 \Phi_\mu(v, \rho) \right] d\rho
\]  

(2)

where

\[
\epsilon = \epsilon^+ + \epsilon^- = \text{total energy of the created pair};
\]

\[
v = \frac{\epsilon}{E}
\]

\[
\rho = (\epsilon^+ - \epsilon^-)/\epsilon = \text{asymmetry coefficient};
\]

The functions \( \Phi_e, \Phi_\mu, \zeta \) can be found in [Koko00].
Direct \((e^+, e^-)\) pair creation by muon

limits

\[
\begin{align*}
\epsilon_{\text{min}} &= 4m_e c^2 \\
\epsilon_{\text{max}} &= E - \frac{3\sqrt{\epsilon}}{4} \ m_\mu c^2 \ Z^{1/3} \\
\rho_{\text{min}} &= 0 \\
\rho_{\text{max}} &= \left[ 1 - \frac{6(m_\mu c^2)^2}{E(E - \epsilon)} \right] \sqrt{1 - \frac{\epsilon_{\text{min}}}{\epsilon}}
\end{align*}
\]
Energetic pairs and truncated energy loss rate

One may wish to take into account separately the high-energy pairs emitted above a given threshold $\epsilon_{cut}$ (miss detection, explicit simulation ...).

Those pairs must be excluded from the mean energy loss count.

$$-\frac{dE}{dx}\bigg|_{\epsilon<\epsilon_{cut}} = n_{at} \int_{\epsilon_{min}}^{\epsilon_{cut}} \epsilon \frac{d\sigma}{d\epsilon} d\epsilon$$

$n_{at}$ is the number of atoms per volume.

Then, the truncated total cross-section for emitting 'hard' pairs is:

$$\sigma(E, \epsilon_{cut} \leq \epsilon \leq \epsilon_{max}) = \int_{\epsilon_{cut}}^{\epsilon_{max}} \frac{d\sigma}{d\epsilon} d\epsilon$$
The muon deflection angle is of the order of:

$$\theta = \frac{mc^2}{E}$$

Above $\sim 1000$ TeV the LPM suppression mechanism may have an effect.
Direct \((e^+, e^-)\) pair creation by muon

number of interactions per cm in Iron. (cut 100 MeV)

![Graph showing the number of interactions per cm in Iron for different processes, including Bremsstrahlung (BREM X-sec), pair production (PAIR X-sec), and MUNU X-sec, as a function of muon kinetic energy (GeV).]
Direct \((e^+, e^-)\) pair creation by muon

1 TeV muon in 10 meter of Fe (field 5 tesla).

direct pair creation only
10 meter of Fe: muons 100 GeV, 1 TeV, 5 TeV.
left: brems only
right: brems + direct pair creation
Direct \( (e^+, e^-) \) pair creation by muon

Energy deposited by high energy muons in a block of Iron

![Graph showing energy deposition vs. peak energy for Geant3 and Geant4 simulations in 2x2x2 m Iron.](image)

- **GeV**
- **E peak (GeV)**
- **Emu (GeV)**

**Legend:**
- **Geant4**
- **Geant3**
Direct \((e^+, e^-)\) pair creation by muon

**energy spectrum of 1 TeV muons after 3 m of iron**

![Graph showing energy spectrum of 1 TeV muons after 3 m of iron]

3 m iron. \(1\) TeV/c
Direct \((e^+, e^-)\) pair creation by muon

References


