

(e^+, e^-) annihilation

(e^+, e^-) annihilation into two photons

$$e^+ + e^- \rightarrow \gamma + \gamma$$

(need two γ for momentum conservation, if the e^- is assumed to be free).

Theoretically, (e^+, e^-) annihilation is related to Compton scattering by crossing symmetry:

- incoming $e^+ \leftrightarrow$ outgoing e^-
- outgoing $\gamma \leftrightarrow$ incoming γ

total cross section per atom

The cross-section formula of Heitler is used [Heitl54]:

$$\sigma(Z, E) = \frac{Z\pi r_e^2}{\gamma + 1} \left[\frac{\gamma^2 + 4\gamma + 1}{\gamma^2 - 1} \ln \left(\gamma + \sqrt{\gamma^2 - 1} \right) - \frac{\gamma + 3}{\sqrt{\gamma^2 - 1}} \right]$$

E = total energy of the incident positron

γ = E/mc^2

r_e = classical electron radius

The cross section **decreases** with increasing E .

The nonrelativistic limit is:

$$\sigma_{nr}(Z, E) \sim \frac{Z\pi r_e^2}{\beta}$$

Mean free path

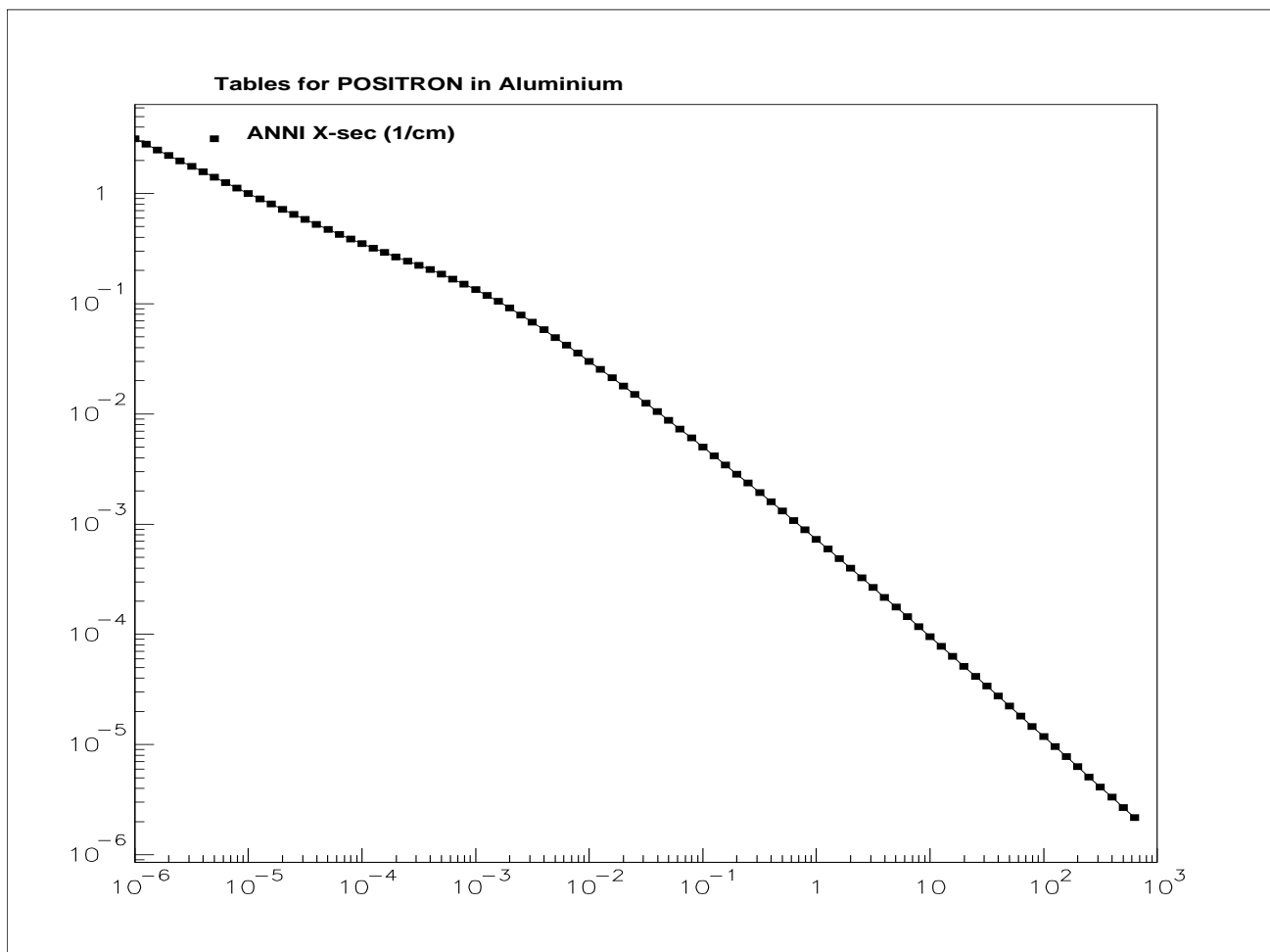
$$\lambda(E) = \left(\sum_i n_{ati} \cdot \sigma(Z_i, E) \right)^{-1}$$

n_{ati} : nb of atoms per volume of the i^{th} element in the material.

At initialization stage, the function `BuildPhysicsTables()` computes and tabulates :

- `crossSectionPerAtom` for all elements
- `meanFreePath` for all materials

number of interactions per cm in Aluminium



positron kinetic energy (GeV)

kinematical limits

$$e^+ e^- \rightarrow \gamma_a \gamma_b$$

The incident e^+ has a total energy: $E = T + mc^2$

The total available energy is: $E_{tot} = E + mc^2 = E_a + E_b$

Let define the ratio of energy transferred to one photon (says γ_a) :

$$\epsilon = \frac{E_a}{E_{tot}} \equiv \frac{E_a}{T + 2mc^2}$$

Energy-momentum conservation gives :

$$\epsilon_{\min} = \frac{E_a^{\min}}{E_{tot}} = \frac{1}{2} \left[1 - \sqrt{\frac{\gamma - 1}{\gamma + 1}} \right] \quad \epsilon_{\max} = \frac{E_a^{\max}}{E_{tot}} = \frac{1}{2} \left[1 + \sqrt{\frac{\gamma - 1}{\gamma + 1}} \right]$$

Therefore the range of ϵ is:

$$\epsilon \in [\epsilon_{\min}, \epsilon_{\max}] \equiv [\epsilon_{\min}, 1 - \epsilon_{\min}]$$

sample the gamma energy

The differential cross-section is :

$$\frac{d\sigma(Z, \epsilon)}{d\epsilon} = \frac{Z\pi r_e^2}{\gamma - 1} \frac{1}{\epsilon} \left[1 + \frac{2\gamma}{(\gamma + 1)^2} - \epsilon - \frac{1}{(\gamma + 1)^2} \frac{1}{\epsilon} \right]$$

The formula can be factorized :

$$\frac{d\sigma(Z, \epsilon)}{d\epsilon} = \frac{Z\pi r_e^2}{\gamma - 1} N f(\epsilon) g(\epsilon)$$

where:

$$N = \ln(\epsilon_{\max}/\epsilon_{\min})$$

$$f(\epsilon) = \frac{1}{N \epsilon}$$

$$g(\epsilon) = \left[1 + \frac{2\gamma}{(\gamma + 1)^2} - \epsilon - \frac{1}{(\gamma + 1)^2} \frac{1}{\epsilon} \right] \equiv 1 - \epsilon + \frac{2\gamma\epsilon - 1}{\epsilon(\gamma + 1)^2}$$

Given 2 random numbers $r_a, r_b \in [0, 1[$:

1. **sample** ϵ from $f(\epsilon) : \epsilon = \epsilon_{\min} \left[\frac{\epsilon_{\max}}{\epsilon_{\min}} \right]^{r_a}$
2. **reject** ϵ if $g(\epsilon) < r_b$

Then the photon energies are :

$$E_a = \epsilon E_{tot} \quad E_b = (1 - \epsilon) E_{tot}$$

The function `PostStepDoIt()` sample ϵ and compute the final kinematic

compute the final kinematic

Lets be θ the angle between the incident e^+ and γ_a .

From the energy-momentum conservation :

$$\cos \theta = \frac{1}{Pc} \left[T + mc^2 \frac{2\epsilon - 1}{\epsilon} \right] = \frac{\epsilon(\gamma + 1) - 1}{\epsilon\sqrt{\gamma^2 - 1}}$$

The azimuthal angle, ϕ , is generated isotropically.

The momentum vector of the photons, \vec{P}_{γ_a} and \vec{P}_{γ_b} are computed from the energy-momentum conservation and transformed to the global coordinate system.

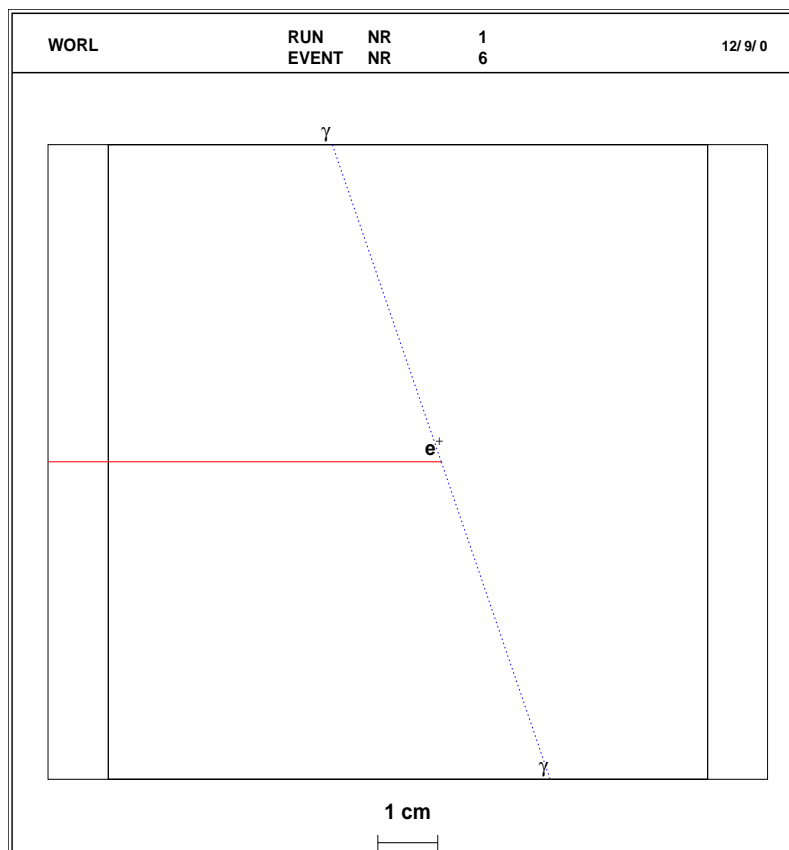
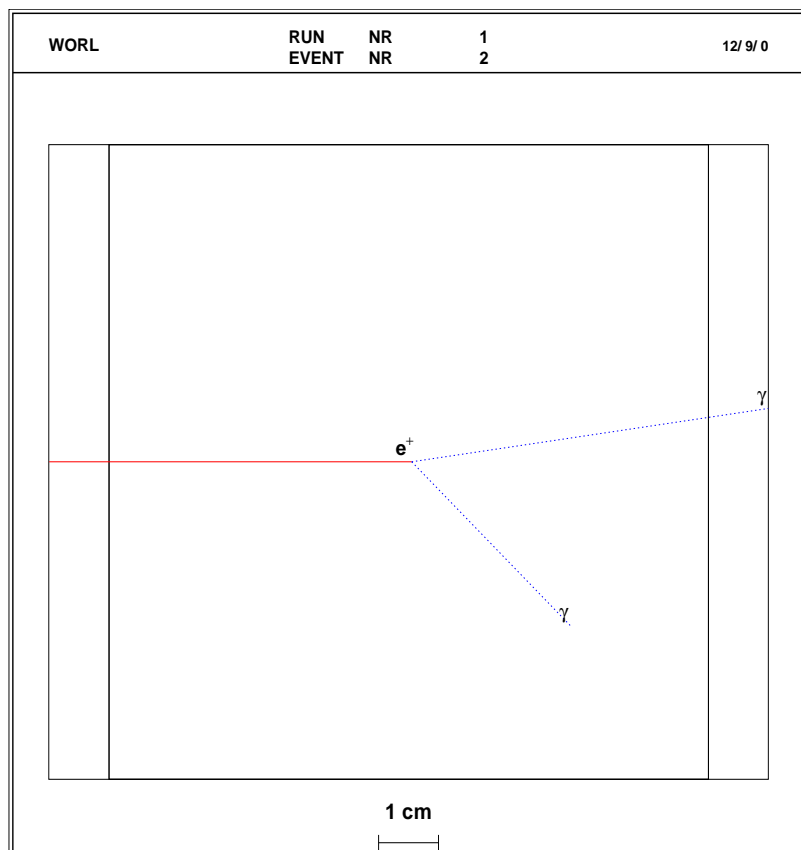
The annihilation **in fly** is not the dominant process. Most of the time the positron comes **at rest** and does a *positronium* with the electron.

The positronium decays in two-photon (in 0.125 *nanosec*) or three-photon state (in 142 *nanosec*.)

The function `AtRestDoIt` treats this case. It generates two photons with energy $E_\gamma = mc^2$. The angular distribution is isotropic.

The (e^+, e^-) can also annihilate in a single photon: the other photon is absorbed by the recoil nucleus. However this mechanism is suppressed by a factor α^4 .

e^+ 30 MeV in 10 cm Aluminium. Annihilation in fly (left), at rest (right).



References

- [Heitl54] W. Heitler. *The Quantum Theory of Radiation*, Clarendon Press, Oxford (1954)