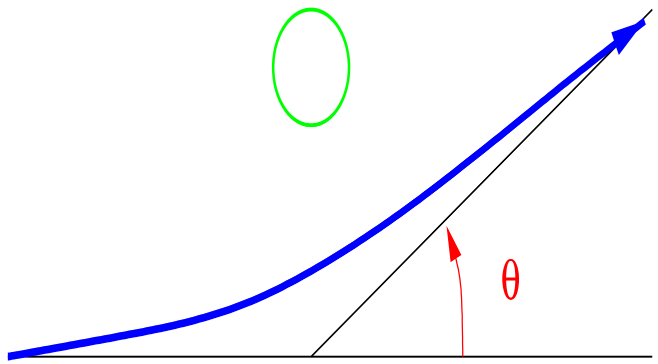
Multiple Coulomb scattering

Single Coulomb scattering

Single Coulomb deflection of a charged particle by a fixed nuclear target.

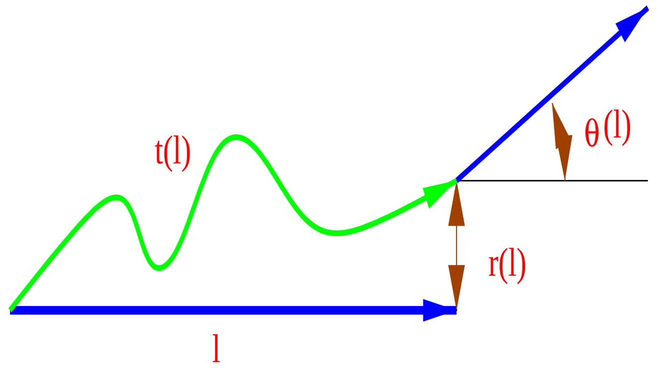


The cross section is given by the Rutherford formula

$$\frac{d\sigma}{d\Omega} = \frac{r_e^2 z_p^2 Z^2}{4} \left(\frac{mc}{\beta p}\right) \frac{1}{\sin^4 \theta/2}$$

Multiple Coulomb scattering

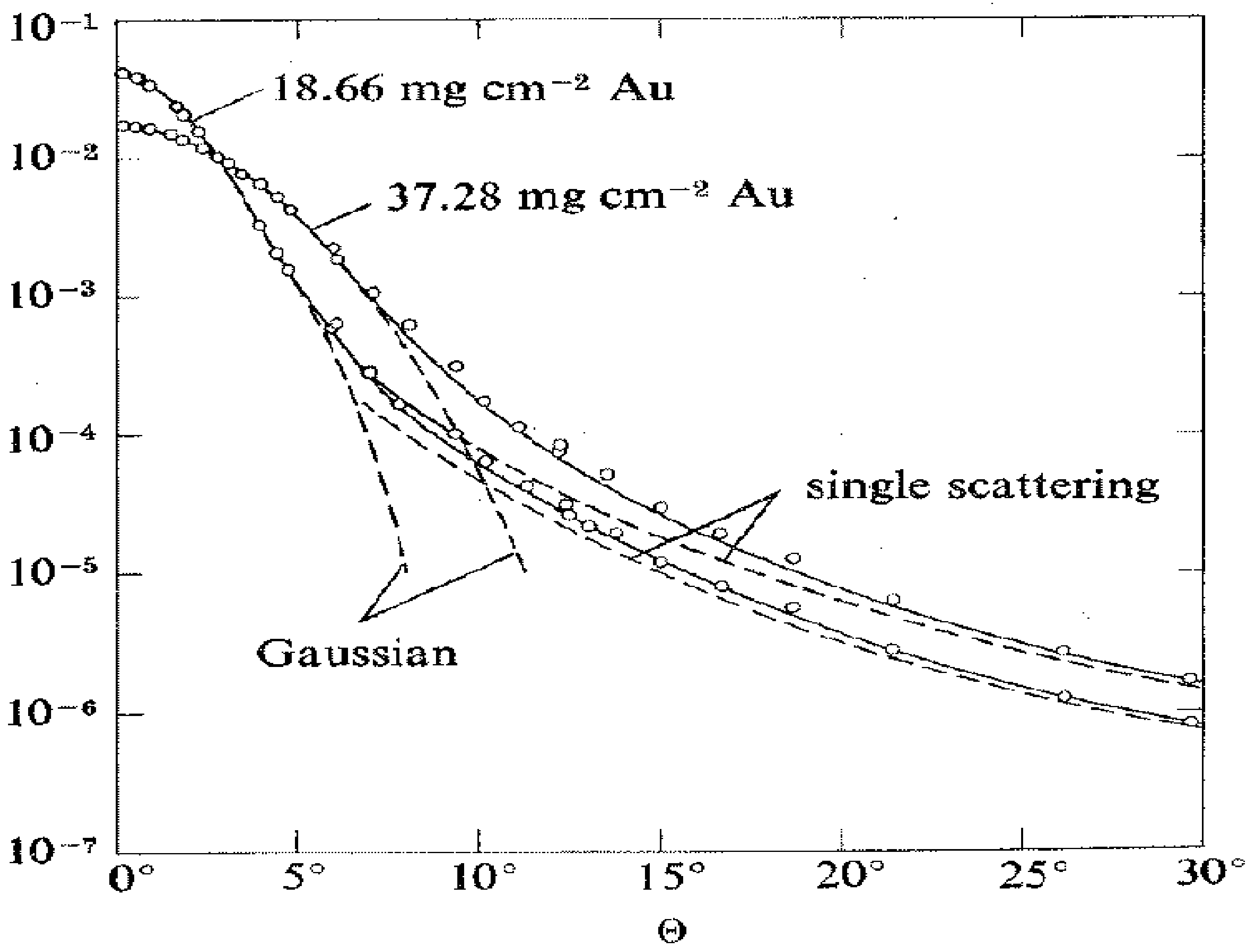
Charged particles traversing a finite thickness of matter suffer repeated elastic Coulomb scattering. The cumulative effect of these small angle scatterings is a net deflection from the original particle direction.



If the number of individual collisions is enough (> 20) the multiple Coulomb scattering angular distribution is gaussian at small angles and like Rutherford scattering at large angles.

The Molière theory reproduces rather well this distribution.

[Mol48, Bethe53]



18.66 mg cm^{-2} Au
37.28 mg cm^{-2} Au
single scattering
Gaussian
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0°
5°
10°
15°
20°
25°
30°
Θ

Gaussian approximation

The central part of the spatial angular distribution is approximately

$$P(\theta)\, d\Omega = \frac{1}{2\pi\theta_0^2}\ \exp\left[-\frac{\theta^2}{2\theta_0^2}\right]\ d\Omega$$

with

$$\theta_0 = \frac{13.6 \text{ MeV}}{\beta\, pc} z \sqrt{\frac{l}{X_0}} \left[1 + 0.038 \ln\left(\frac{l}{X_0}\right)\right]$$

where l/X_0 is the thickness of the medium measured in radiation lengths X_0.

This formula of θ_0 is from a fit to Molière distribution. It is accurate to $\leq 10\%$ for $10^{-3} < l/X_0 < 10^2$

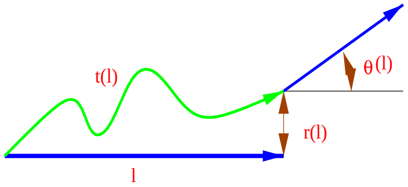
note: the appearance of X_0 in the formula is only for convenience.

Others formulas for θ_0 have been developed, starting from the Molière theory. [Lynch91]

related quantities

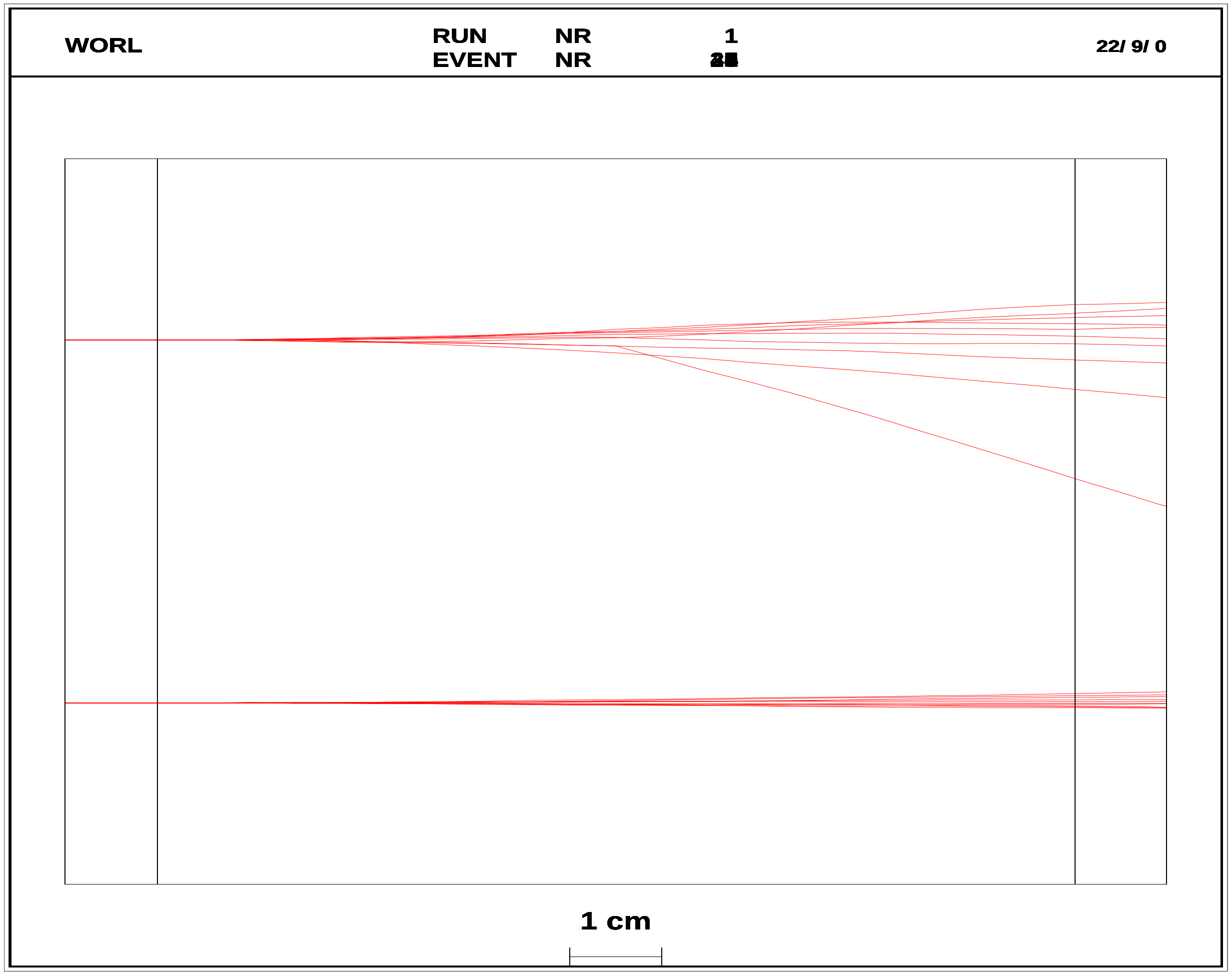
- lateral displacement $r(l)$
- true (or corrected) path length $t(l)$
- projected angular deflection $\theta_{proj}(l)$

they are correlated random variables, for instance needed in Monte Carlo simulation.



Energy dependence

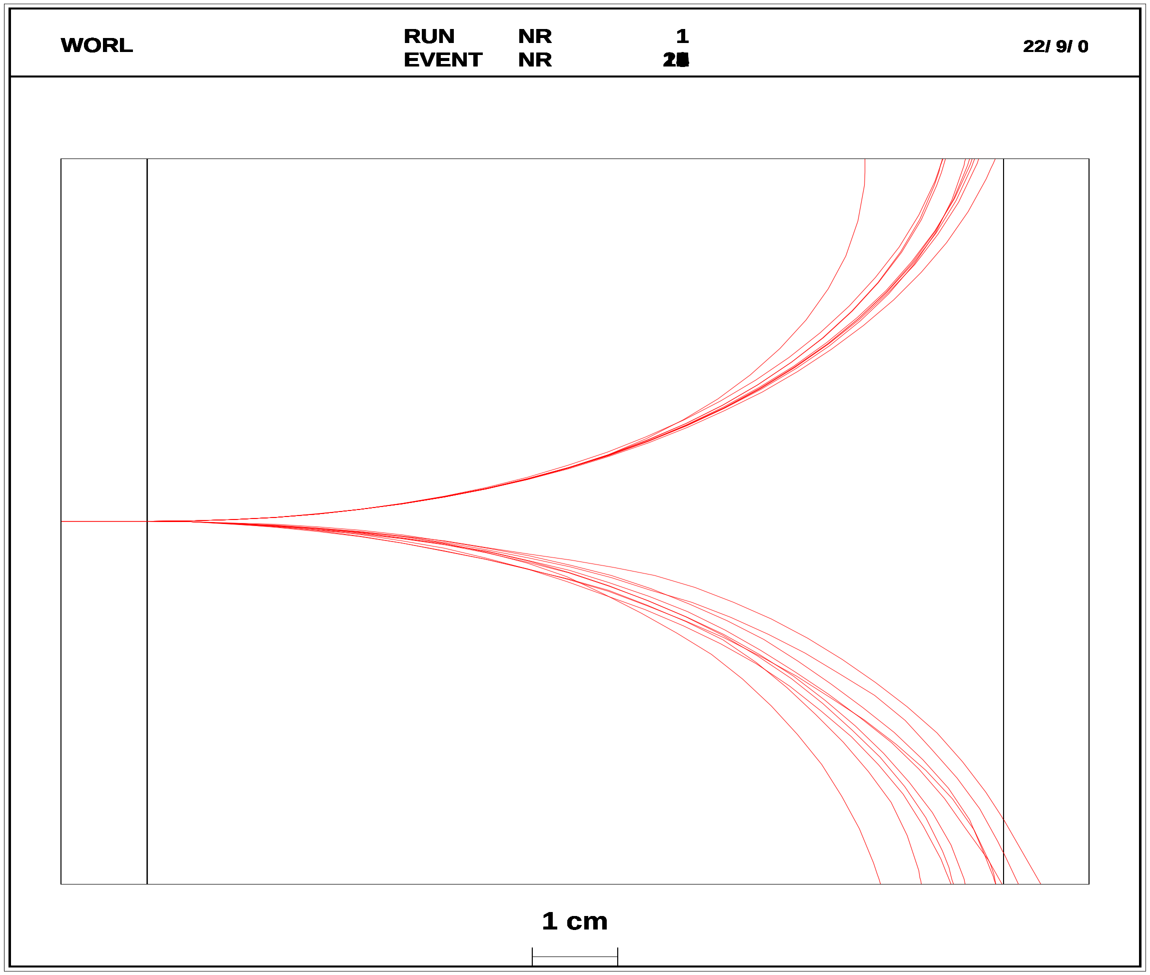
10 π^+ of 200 MeV and 1 GeV crossing 10 cm of Aluminium.



10 cm of Aluminium. Field 5 tesla.

top: 10 e^- (300 MeV): energy loss fluctuations only (no muls)

bottom: 10 e^+ (300 MeV): multiple scattering only (no eloss fluct)



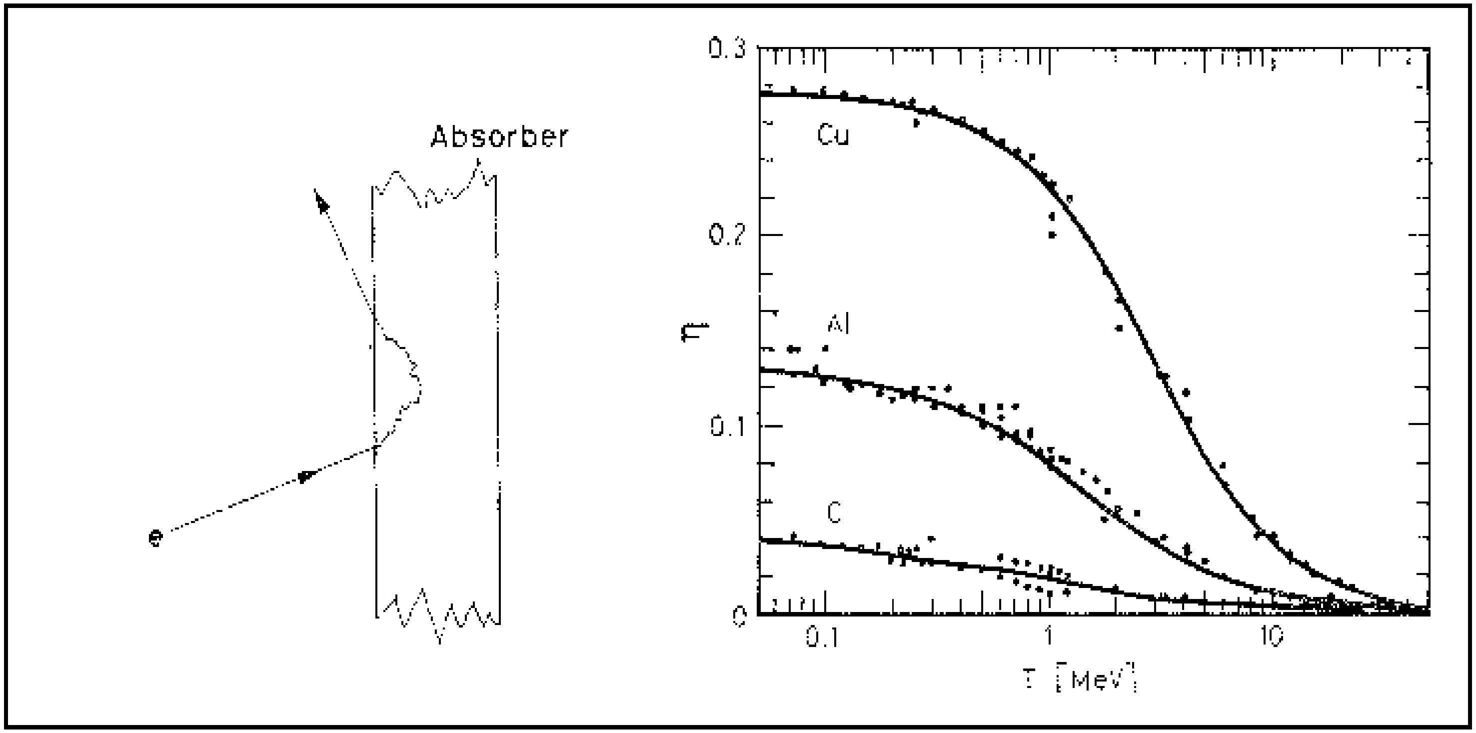
Others models for simulation

Several models of multiple Coulomb scattering simulation algorithms have been proposed, not necessarily based on the Molière theory. (See the references in [PDG00])
For instance :

- J.M. Fernandez-Verea et al. : a ”mixed” (detailed + condensed) model. [Fer93]
- L.Urban : a condensed model based on Lewis theory.[Urb00]

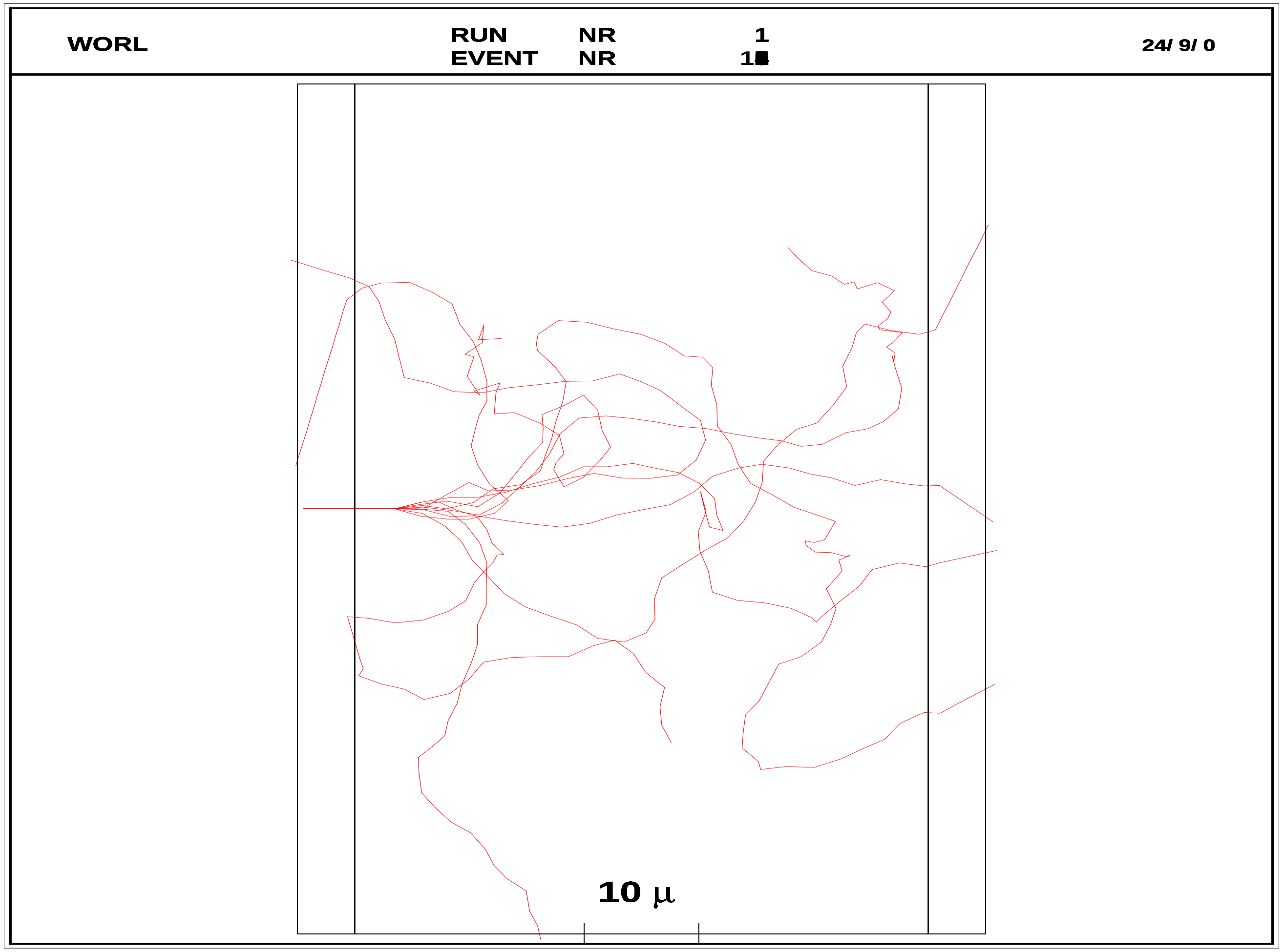
backscattering of low energy electrons

Because of its small mass, electron can have large deflection by scattering from nuclei.
For low energy incident electron beam, the ratio of electrons which are backscattered out of the detector may be important (*albedo*).



albedo : The incident beam is 10 electrons of 600 keV entering in 50 μm of Tungsten.

4 electrons are transmitted, 2 are backscattered.

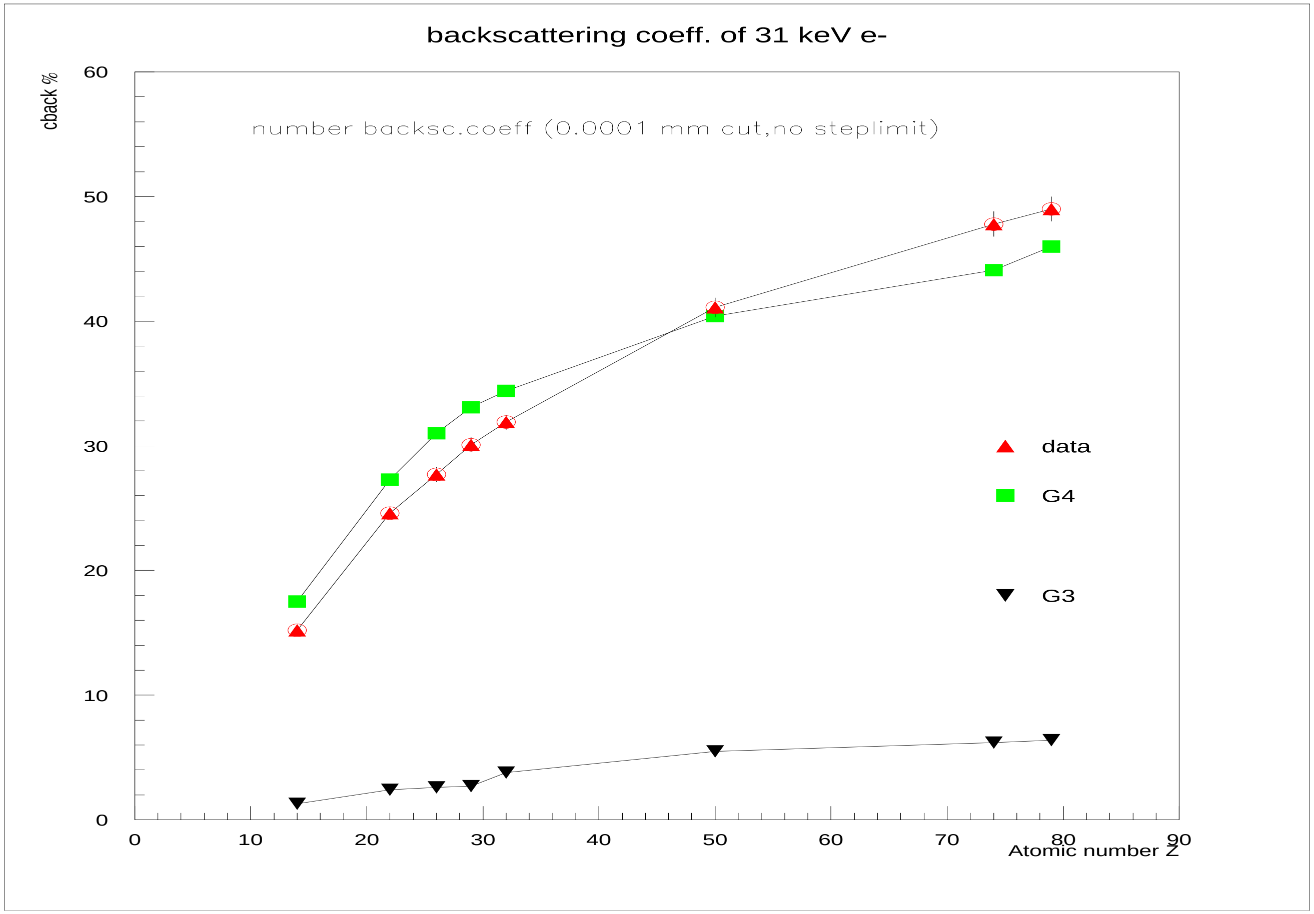


low energy simulation, electron/positron backscattering not a trivial task for a code designed for high energy physics applications in mind

e.g. GEANT3 is not able to reproduce the experimental data, - sim.results are far from data and they are unstable, a small change in the cut or max.step limitation triggers big changes in the results

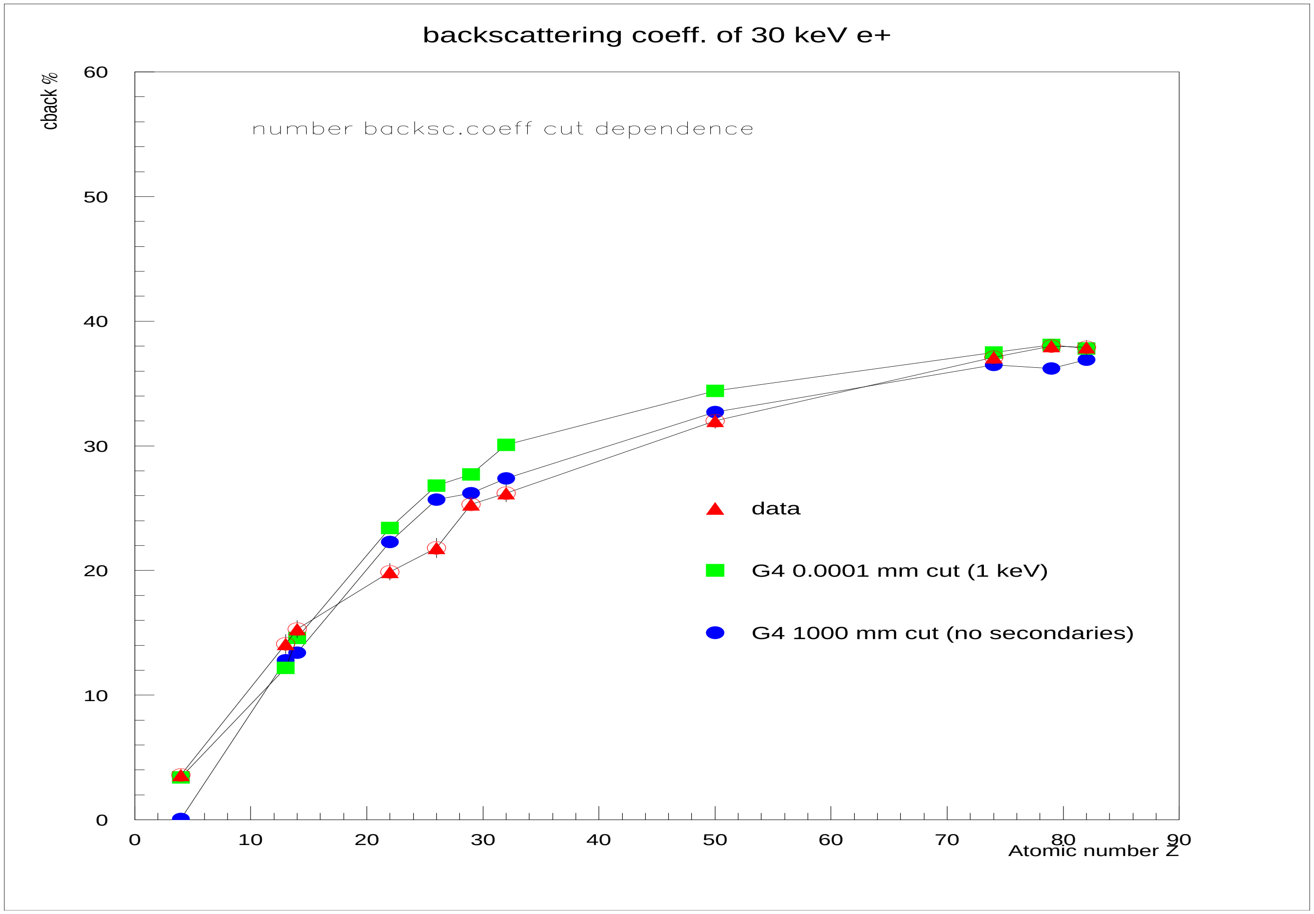
GEANT4 can do the job ! (some other simulation codes can simulate backscattering, but these are 'microscopic' or 'mixed' MC codes ...)

Some results follow ....



backscattering of 31 keV e- from diff. targets(thick)

→ G4 = data , G3 ≪ data



backscattering of 30 keV e+ from diff. targets(thick)

→ G4 = data and the simulation does not depend on the cut!

References

[Mol48] Molière, Z. Naturforsch. 3a, 78 (1948)

[Bethe53] H.A.Bethe, Phys. Rev. 89, 1256 (1953)

[Lynch91] G.R.Lynch and O.I.Dahl, NIM B58,6 (1991)

[Fer93] J.M. Fernandez-Verea et al. NIM B73, 447 (1993)

[Urb00] L. Urban http://wwwinfo.cern.ch/asd/geant4/geant4.html

[PDG00] D.E. Groom et al. Particle Data Group . Rev. of Particle Properties. Eur. Phys. J. C15,1 (2000)
http://pdg.lbl.gov/