Photoelectric absorption

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A bound electron can absorb completely the energy of a photon :

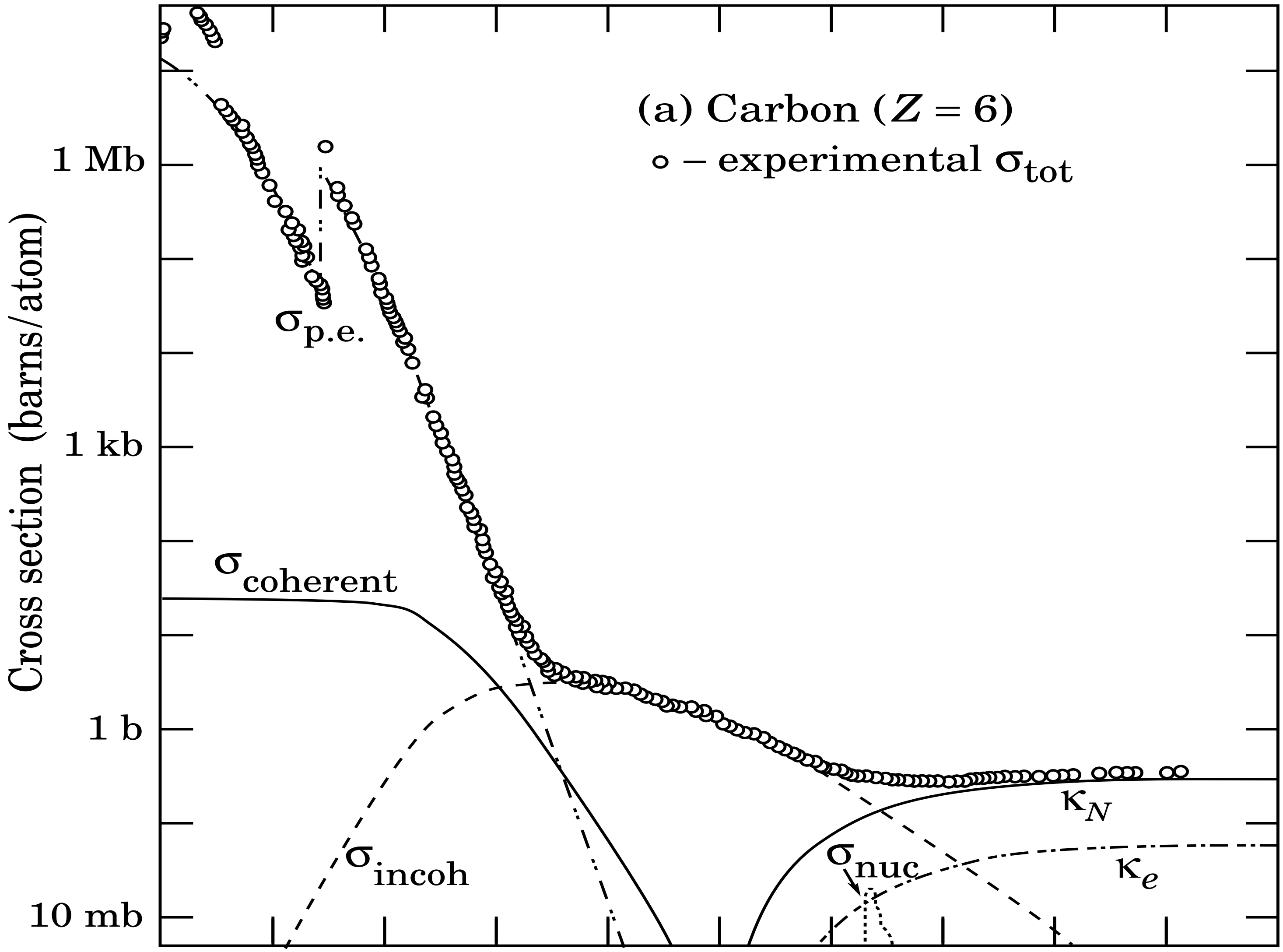
$$\gamma + atom \rightarrow atom^{+} + e^{-}$$

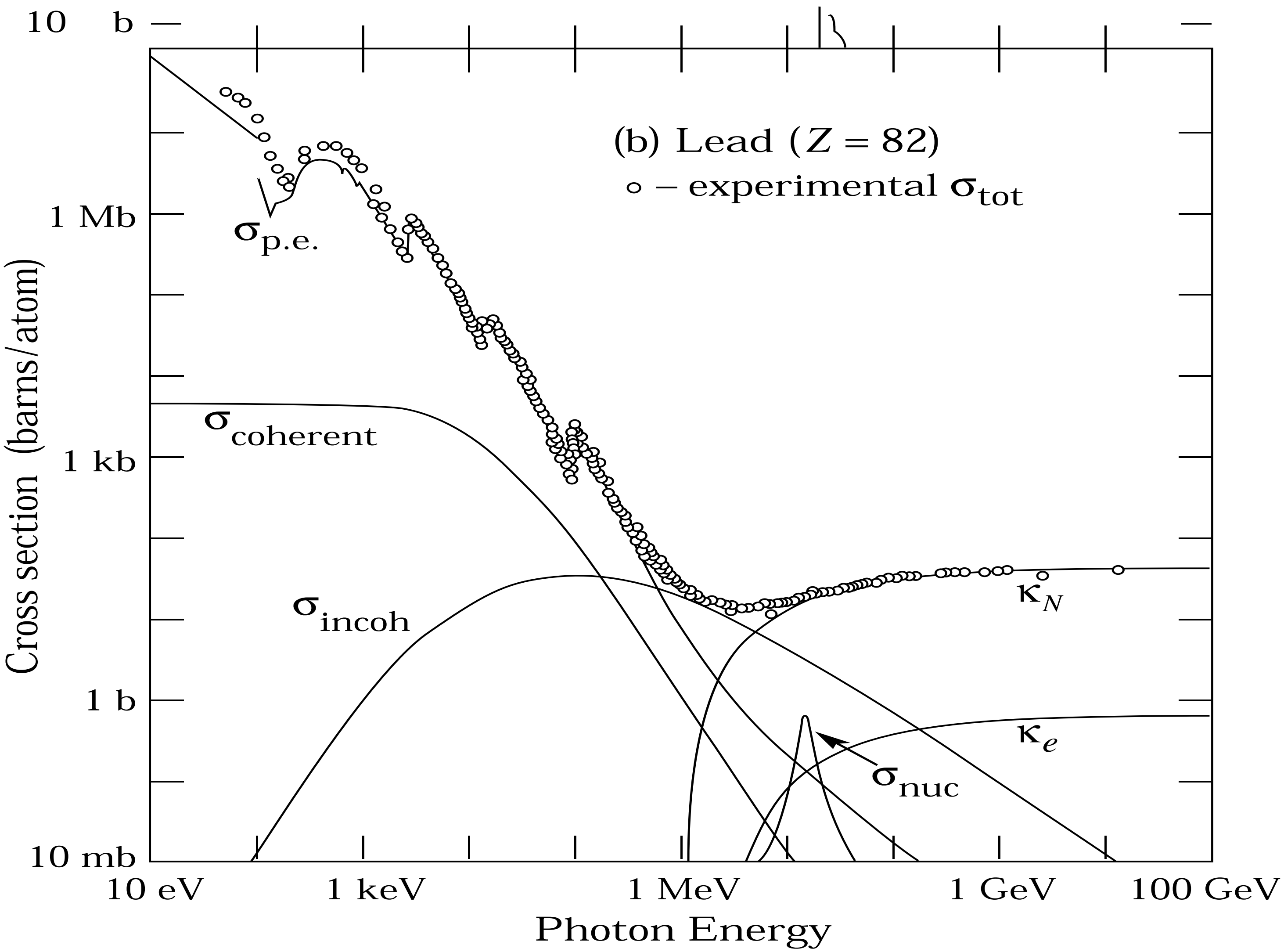
The electron is ejected with kinetic energy $T = E_\gamma - B_s$.
(E_γ: energy of the incident photon, B_s: binding energy of the corresponding subshell). The nucleus absorbs the recoil momentum.

The cross section per shell can be parametrized [Biggs87] :

$$\sigma_s = r_e^2\, \alpha^4\, Z^5\, f\left(\frac{1}{E_\gamma^{a(E_\gamma)}}\right)$$

with f : nonsimple function of $1/E_\gamma$, and $1 \leq a(E_\gamma) \leq 4$.


(a) Carbon ($Z = 6$)
○ – experimental σ_{tot}
Cross section (barns/atom)
1 Mb
1 kb
1 b
10 mb
$\sigma_{p.e.}$
$\sigma_{coherent}$
σ_{incoh}
σ_{nuc}
κ_N
κ_e


(b) Lead ($Z = 82$)
○ – experimental σ_{tot}
$\sigma_{p.e.}$
$\sigma_{coherent}$
σ_{incoh}
κ_N
κ_e
σ_{nuc}
Cross section (barns/atom)
10 b
1 Mb
1 kb
1 b
10 mb
10 eV
1 keV
1 MeV
1 GeV
100 GeV
Photon Energy

The total cross section has discontinuities at $E_\gamma = B_s$ (absorption edge).

There are several parametrizations and tables of the cross sections. See [Cullen97, Biggs87].

If $E_\gamma > B_K$ the absorption occurs mainly on the K-shell (80% of the cases).

The electron is emitted forward in the direction of the incident photon at high E_γ, and perpendicular to the photon at low E_γ [Sauter31, Gavri61].

Following the photoabsorption in the K-shell, characteristic X-rays or Auger electrons are emitted [Perkin91].

Cross section per atom above 50 keV in GEANT4

E_γ = incident gamma energy, and $\epsilon = E_\gamma / m_e c^2$

The total cross-section per atom is parameterised as [GEANT3]:

$$\sigma(Z,\epsilon) = \frac{Z^\alpha}{\epsilon^\beta} F(Z,\epsilon)$$

where α and β are results of a fit

$$\begin{aligned} F(Z,\epsilon) \quad = \quad & p_{1K}/Z + p_{2K}/\epsilon + p_{3K} + p_{4K}Z + p_{5K}\epsilon + p_{6K}Z^2 + \\ & p_{7K}Z\epsilon + p_{8K}\epsilon^2 + p_{9K}Z^3 + p_{10K}Z^2\epsilon + p_{11K}Z/\epsilon^2 + \\ & p_{12K}\epsilon^3 \end{aligned}$$

and similar formulas for L_1 and L_2 shells.

The fit was made over 301 data points chosen between:

$$5 \leq Z \leq 100 \quad \text{and} \quad E_\gamma \in [10\ keV\ , 50\ MeV]$$

The accuracy of the fit is estimated to be:

$$\frac{\Delta\sigma}{\sigma} \leq \begin{cases} 25\% & \text{near to the peaks} \\ 10\% & \text{elsewhere.} \end{cases}$$

The binding energy of the inner shells have been parameterised as:

$$B_i(Z) = Z^2(a_i + b_i Z + c_i Z^2 + d_i Z^3)$$

where $i = K, L_1, L_2$, and the constants a_i, b_i, c_i, d_i are tabulated inside dedicated functions.

Cross section per atom below 50 keV

We use a parametrisation of the photoabsorption cross section proposed by Biggs and al. [Biggs87] :

$$\sigma(Z, E_\gamma) = \frac{a(Z, E_\gamma)}{E_\gamma} + \frac{b(Z, E_\gamma)}{E_\gamma^2} + \frac{c(Z, E_\gamma)}{E_\gamma^3} + \frac{d(Z, E_\gamma)}{E_\gamma^4} \qquad (1)$$

The coefficients a, b, c, d are fitted with experimental data by the least square method separately in each energy interval [Grichi94].

As a rule, the interval borders are equal to the corresponding photoabsorption edges.

Mean free path

$$\lambda(E_\gamma) = \left(\sum_i n_{ati} \cdot \sigma(Z_i, E_\gamma) \right)^{-1}$$

n_{ati} : nb of atoms per volume of the i^{th} element in the material.

At initialization stage, the function `BuildPhysicsTables()` computes and tabulates :

- crossSectionPerAtom for all elements
- meanFreePath for all materials

The cross section and mean free path can be tabulated only above 50 keV. Below this, they are too discontinue: they are recomputed 'on fly' from formula 1.

final state

choose an Element : the binding energy of the shells depend of Z_i. In a compound material one choose randomly an Element according :

$$Prob(Z_i, E_\gamma) = \frac{n_{ati}\sigma(Z_i, E_\gamma)}{\sum_i [n_{ati} \cdot \sigma_i(E_\gamma)]}$$

final state : the simulation is presently rather crude.
A quanta can be absorbed if $E_\gamma > B_{shell}$. The shell energies are taken from `G4AtomicShells` data. One choose the closest atomic shell available.

The photoelectron is emitted with kinetic energy:

$$T_{photoelectron} = E_\gamma - B_{shell}(Z_i)$$

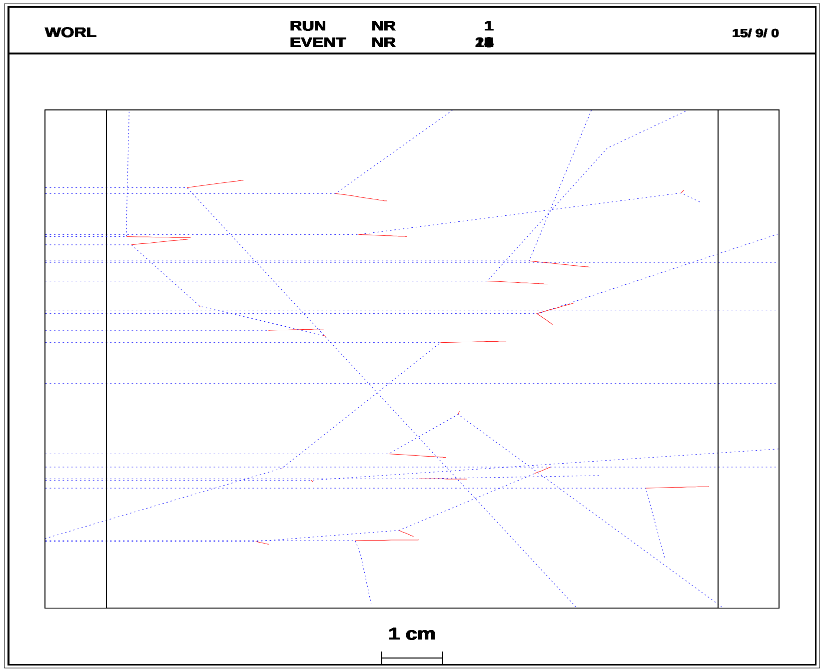
The electron has the same direction as the incident gamma.

attenuation

$$\sigma_{tot} = \sigma_{pair} + \sigma_{comp} + \sigma_{phot} + \sigma_{rayl} \quad \longrightarrow \mu = n_{at}\,\sigma_{tot}$$

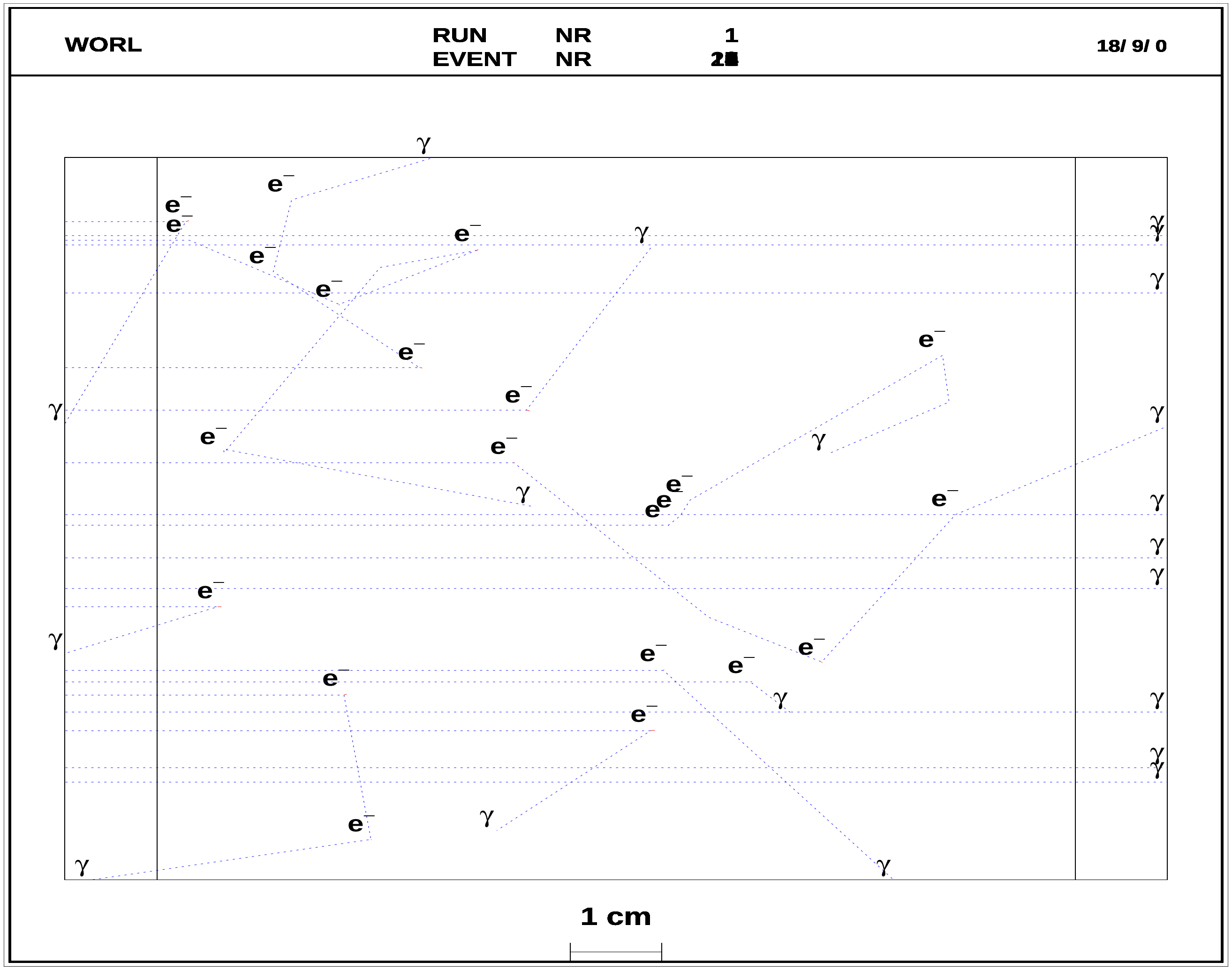
A beam of monoenergetic photons is attenuated in intensity (not in energy) according : $I(x) = I(0)\exp(-\mu\, x) = I(0)\exp(-x/\lambda)$

Below : 20 photons, 5 MeV, entering 10 cm of Al. 4 exit unaltered.

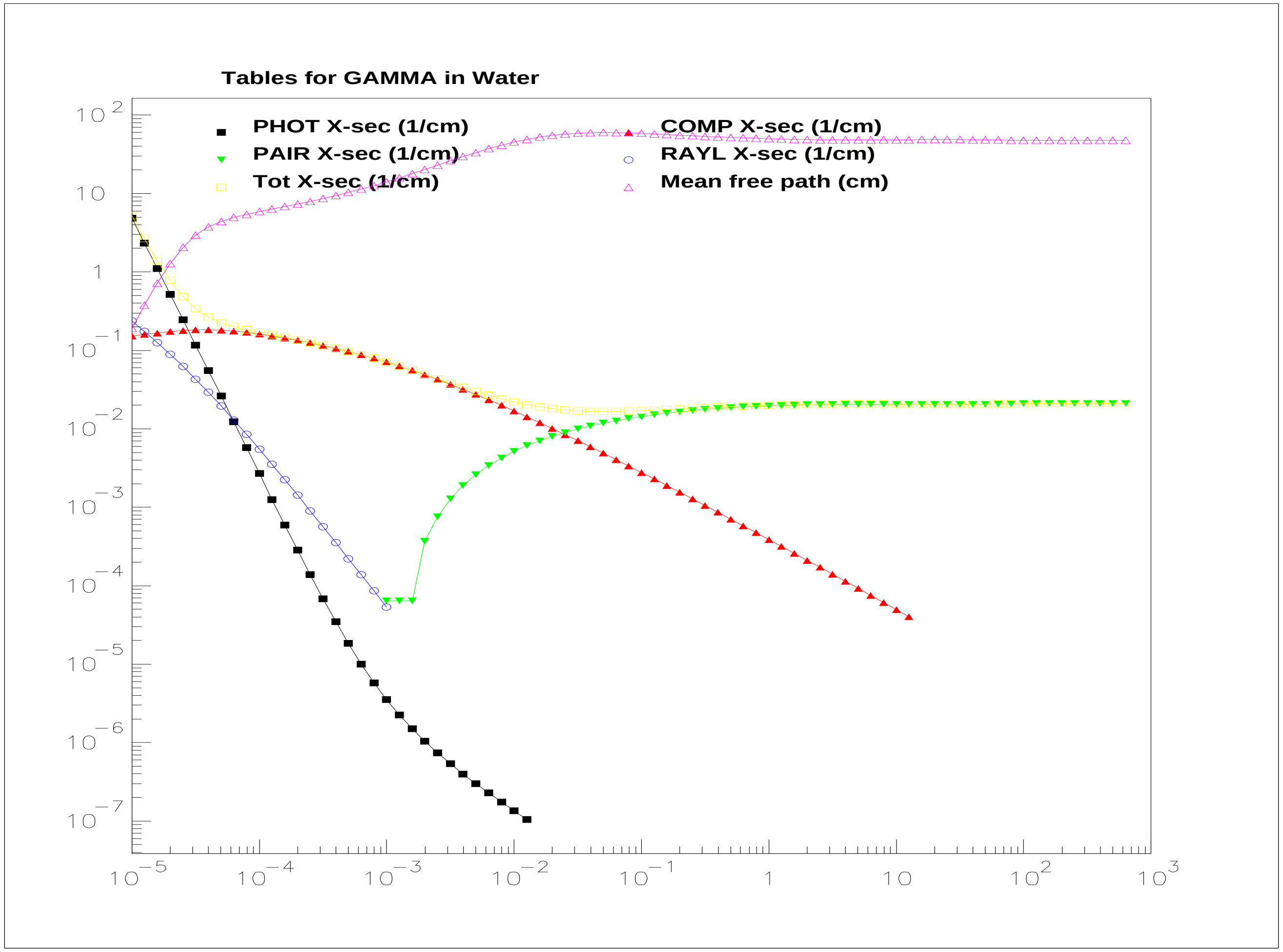


20 photons, 400 keV, entering 10 cm of water.

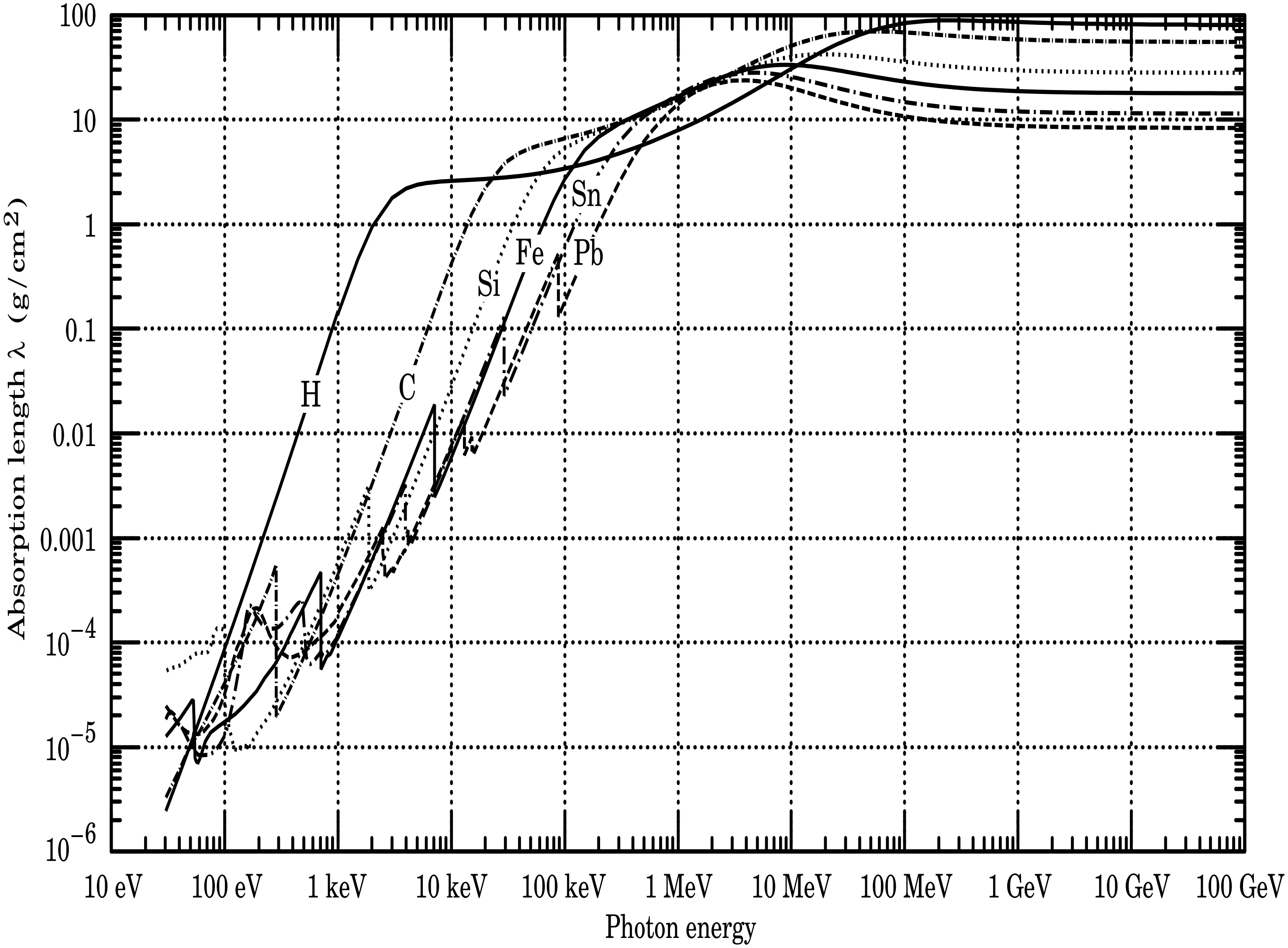
(compare with e^- and protons)



Macroscopic cross sections for photon in water. (⟶ mean free path)



photon energy (GeV)


Absorption length λ (g/cm²)
100
10
1
0.1
0.01
0.001
10^{-4}
10^{-5}
10^{-6}
10 eV
100 eV
1 keV
10 keV
100 keV
1 MeV
10 MeV
100 MeV
1 GeV
10 GeV
100 GeV
Photon energy
H
C
Si
Fe
Sn
Pb

References

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J.H.Hubbell et al. Rad. Phys. Chem. vol50, 1 (1997)

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