Lepton and gamma nuclear reactions

J.P. Wellisch, CERN/EP,
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Outline

- Partices treated
- Cross-section calculations
  - The modeling
  - Classes exposed to users
  - Restrictions of applicability
- Final state generation
  - The modeling
  - Restrictions of applicability
  - Classes exposed to users

Particles treated

- Gamma - yes
- Electron - yes
- Positron - yes
- Muons - yes
- Neutrinos – not at present

Gamma nuclear reaction cross-sections

- Modeling the following regions:
  - GiantDipoleResonance regime
    - O(100MeV)
  - Roper regime
    - the intermediate energy desert
  - Delta regime
    - O(1-2GeV)
  - Reggeon-pomeron regime

GDR regime

- GDR from power law (CHIPS), and nuclear barrier reflection function (which looks like a threshold)

$$GDR(e,p,b,c,s) = T(e,b,s)\exp(c - pe)$$

$$T(e,b,s) = 1 \big/ 1 + \exp((b-e)/s)$$

- Here e is log(E_g).
- Parameters p, b, c, s are tuned on experimental data (From He to U), H and d are treated as special cases

Delta isobar region

- The delta isobar region is modeled like a Breit Wiegner function plus production threshold:

$$\Delta(e,d,f,g,r,q) = \frac{d \cdot T(e,f,g)}{1 + r \cdot (e-q)^2}$$

- Here q can be looked at as the position of the delta, and r as the inverse width
- The parameters are tuned on experimental data as a function of log(A)

Roper region

- This regime was parametrized using the same functional form, dropping the pion threshold factor

$$Tr(e, v, w, u) = \frac{v}{1 + w \cdot (e - u)^2}$$

Reggeon-Pomeron region

- In the reggeon-pomeron region, we use

$$RP(e,h) = h \cdot T(e,7.0,0.2) \cdot (0.0116 \cdot \exp(0.16 \cdot e) + 0.4 \cdot \exp(-0.2 \cdot e)), with$$

$$h = A \cdot \exp(-\log(A) \cdot (0.885 + 0.0048 \cdot \log(A)))$$

Interface classes to gamma nuclear reaction cross-sections

- There is exactly one interface class:
- G4PhotoNulearCrossSection

Implementation restrictions:

- None.
- Details currently being prepared for publication in a refereed journal.

Chiral Invariant Phase-space Decay: gamma nuclear reactions

- A quark level 3-dimensional event generator for fragmentation of excited hadronic systems into hadrons.
- Based on of asymptotic freedom.
- Local chiral invariance restoration lets us consider quark partons massless. We can fold the invariant phase-space distribution of quark partons with the quark exchange (fusion) probability of hadronization.
- The only non-kinematical concept used is that of a temperature of the hadronic system (Quasmon).

Vacuum CHIPS

- This allows to calculate the decay of free excited hadronic systems:
- In an finite thermalized system of N partons with total mass M, the invariant phase-space integral is proportional to M^{2N-4}, and the statistical density of states is proportional to $e^{-M/T}$. Hence we can write the probability to find N partons with temperature T in a state with mass M as $dW \propto M^{2N-4} e^{-M/T} dM$
- Note that for this distribution, the mean mass square is $\langle M^2 \rangle = 2N(2N-2)T^2$

Vacuum CHIPS

- We use this formula to calculate the number of partons in an excited thermalized hadronic system, and obtain the parton spectrum

$$\frac{dW}{kdk} \propto \left(1 - \frac{2k}{M}\right)^{N-3}$$

- To obtain the probability for quark fusion into hadrons, we can now compute the probability to find two partons with momenta q and k with the invariant mass μ.

$$P(k, M, \mu) = \int \left(1 - \frac{2q}{M\sqrt{1-2k/M}}\right)^{N-4} \times \delta\left(\mu^2 - \frac{2kq(1-\cos\theta)}{\sqrt{1-2k/M}}\right) q\,dq\,d\cos\theta$$

Vacuum CHIPS

- Using the delta function to perform the integration and the mass constraint, we find the total kinematical probability of hadronization of a parton with momentum k into a hadron with mass μ:

$$\frac{M-2k}{4k(N-3)}\left(1-\mu^2/2kM\right)^{N-3}$$

- Taking into account spin and quark content of the final state hadron adds (2s+1) and a combinatorial factor.
- At this level of the language, CHIPS can be applied to p-pbar annihilation

Nuclear CHIPS

- In order to apply CHIPS for an excited hadronic system within nuclei, we have to add parton exchange with nuclear clusters to the model
- The kinematical picture is, that a color neutral Quasmon emits a parton, which is absorbed by a nucleon or a nuclear cluster. This results in a colored residual Quasmon, and a colored compound.
- The colored compound then decays into an outgoing nuclear fragment and a 'recoil' quark that is incorporated by the colored Quasmon.

Nuclear CHIPS

- Applying mechanisms analogue to vacuum CHIPS, we can write the probability of emission of a nuclear fragment with mass μ as a result of the transition of a parton with momentum k from the quasmon to a fragment with mass μ' as:

$$P(k,\mu',\mu) = \int \left(1 - \frac{2(k-\Delta)}{\mu' + k(1-\cos\theta_{kq})} \right)^{n-3} \frac{\mu'(k-\Delta)}{2\left[\mu' + k(1-\cos\theta_{kq})\right]^2} \, d\cos\theta_{kq}$$

- Here, n is the number of quark-partons in the nuclear cluster, and Δ is the covariant binding energy of the cluster, and the integral is over the angle between parton and recoil parton.

Nuclear CHIPS

- To calculate the fragment yields it is necessary to calculate the probability to find a cluster of ν nucleons within a nucleus. We do this using the following assumptions:
  - A fraction $\varepsilon 1$ of all nucleons is not clustering
  - A fraction $\varepsilon 2$ of the nucleons in the periphery of the nucleus is clustering into two nucleon clusters
  - There is a single clusterization probability ω
- and find, with a being the number of nucleons involved in clusterization

$$P_\nu = \frac{C_\nu^a \omega^{\nu-1}}{(1+\omega)^{a-1}}$$

Nuclear CHIPS

- At this level of the language, CHIPS can be applied photo-nuclear and electro-nuclear reactions.

Interface classes to gamma nuclear final state modeling

- There is exactly one:
- G4GammaNuclearReaction

- Both cross-section and final state model are made to be registered with G4PhotoNuclearProcess

Implementation restrictions

- Currently valid up to gamma energies of 3 GeV.
- Above, quark gluon string model can be used.
- More work on refining the tuning of the pomeron vertex parameters in qgs model will be scheduled, as user community grows.

Electro-nuclear scattering: reaction cross-sections

- Based on Fermi's method of equivalent photons, as developed by Weitzsaecker and Williams:

$$dN_\gamma = -\frac{2\alpha}{\pi}\log(y)d\log(y),\ y = {}^{\nu}\!/_{E_e}$$

- Folding this flux with the gamma reaction cross-section (as described above) and integrating the gamma spectrum, we obtain:

$$\sigma(eA \rightarrow X) = \log(E_e)\int\frac{2\alpha}{\pi}\sigma_{\gamma A}(\nu)d\log(\nu) - \int\frac{2\alpha}{\pi}\log(\nu)\sigma_{\gamma A}(\nu)d\log(\nu)$$

Electro-nuclear scattering: reaction cross-sections

- The integrals have been tabulated into look-up tables with linear interpolation, for a set of nuclei.

Interface classes to electro nuclear cross-sections

- There is one interface class
- G4ElectroNuclearCrossSection

Restrictions

- The modeling assumptions in the equivalent photon spectrum.
- The DIS part (small) of the cross-section is neglected
- To smoothen the electro-nuclear cross-section at 2 GeV, the PR term used was

$$RP(e,h) = h \cdot (0.0116 \cdot \exp(0.16 \cdot e) + 1.0 \cdot \exp(-0.26 \cdot e))$$

Electro-nuclear scattering: final state modeling

- The formulas presented in the cross-section section can be used to calculate the probability distribution of the equivalent photons.
- This distribution is samples to calculate the energy transfer of the nuclear reaction
- The energy is assumed to be transferred by gamma exchange, and the models for gamma nuclear reaction (CHIPS) are used.

Interface classes to electro nuclear modeling

- The process classes are G4ElectronNuclearProcess, and G4PositronNuclearProcess
- The final state model class is G4ElectroNuclearReaction

Implementation restrictions

- The energy transfer is at present limited to 3 GeV per collision.
- The DIS part is neglected

Muon nuclear reactions

- The muon nuclear cross-section has some relevance at energies larger 10 GeV, and relatively high energy transfers.
- It is about 10% of the total energy loss for 10TeV muons.

The differencial cross-section

- The Borog Petrukhin formula is used for the cross-section differential in transferred energy

$$\sigma(E,\nu) = \Phi(E,y)\Psi(\nu);$$

$$\Psi(\nu) = \frac{\alpha}{\pi}\frac{A_{eff} N_{AV}}{A}\sigma_{\gamma N}(\nu)\frac{1}{\nu};$$

$$\Phi(E,y) = y - 1 + \left[1 - y + \frac{y^2}{2}\left(1 + \frac{2\mu^2}{\Lambda^2}\right)\right]\ln\frac{\frac{E^{2(1-y)}}{\mu^2}\left(1 + \frac{\mu^2 y^2}{\Lambda^2(1-y)}\right)}{1 + \frac{Ey}{\Lambda}\left(1 + \frac{\Lambda}{2M} + \frac{Ey}{\Lambda}\right)};$$

$$A_{eff} = 0.22A + 0.78A^{0.89};$$

$$\sigma_{\gamma N}(\nu) = \left(49.2 + 11.1\ln(\nu) + 151.8/\sqrt{\nu}\right)\cdot 10^{-30}\,cm^2$$

Energy transfer, cont.

- The transferred energy is then found by sampling this cross-section

Total reaction cross-section

- This cross-section is derived from the formula given by integration over all energy transfers of the cross-section differential in energy transfer given at the next slide.

Muon scattering angle

- The scattering angle of the muon can be calculated by sampling the cross-section differential in momentum transfer for fixed energy transfer:

$$\frac{d\sigma}{dt} \propto \frac{(1 - t/t_{\max})}{t(1 + t/\nu^2)(1 + t/m_0{}^2)} \left[(1 - y)(1 - t_{\min}/t) + y^2/2\right]$$

- T_max and t_min are defined by kinematic constraints, and m_0 is a phenomenological parameter of the inelastic form-factor

Hadronic vertex

- With the formalism described, we have a complete description of the leptonic vertex, including the exchange particle.
- In the context of the VMD, this exchange gamma is re-interpreted as a rho (or omega) meson, to be absorbed by the nucleus.
- In the present implementation, the hadronic vertex is modeled in rough approximation, using the parametrized models for meson scattering. This is to be replaced with CHIPS/QGS in due course.

Interface classes

- Both cross-section and final state modeling are part of the interface of G4MuNuclearInteraction.
- G4MuNuclearInteraction is a discrete process.